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A digraph-based hexahedral meshing method for coupled quasi-polycubes

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ABSTRACT

Sub-mapping is recognized as the most commonly used method for generating structured hexahedral mesh which has become a prior choice for mechanical simulation. However, volume decomposition process of sub-mapping, due to the diversity of entity structures, still lacks support from stable algorithm. Moreover, as a preprocessing step of sub-mapping, morphing process sometimes leads to unsatisfactory mesh quality. In order to improve stability and meshing quality of sub-mapping, a digraph-based structured hexahedral meshing method for coupled quasi-polycubes is proposed in this paper. The method avoids the processes of volume decomposition and morphing. At first, an initial manual decomposition is carried out. The input model is decomposed into a number of sub-entities with respect to which initial digraphs are built. Secondly, virtual edges and vertices are constructed by means of mean value interpolation and then added into its corresponding digraph, so as to build connections between subgraphs within one digraph. Thirdly, face sheets are obtained by intersecting sub-entities with one another. Vertices and edges of the face sheets are added into their corresponding sub-entities' digraphs to establish connections between digraphs of sub-entities. Finally, a linear system is constructed according to the solution of which the whole model is meshed. Experiments revealed in this paper demonstrate that the proposed method is applicable for all-structured hexahedral mesh generation of coupled quasi-polycubes.

1. Introduction

Mesh generation is a crucial input for finite element method and finite difference method, thus the precondition of simulation. Accuracy and duration of simulation are highly dependent on the average and the worst quality of the mesh. Because of providing better element stiffness matrix, high quality meshes always lead to a quicker and more accurate solution. The quality of hexahedral mesh is much better than tetrahedral. In order to achieve the same level of accuracy, tetrahedral mesh typically require four to ten times more elements and much longer duration than a hexahedral mesh [1,2]. Due to the higher quality of structured hexahedral mesh than unstructured hexahedral mesh and aligning with the boundary, structured hexahedral mesh can significantly reduce the processing time and simplify parallel computing, and consequently becomes an ideal choice for mesh generation.

As far as concerned, no valid solution is able to generate all-structured hexahedral mesh stably and automatically. In actual projects, mesh generation has always been done manually to ensure meshing quality. Therefore plenty of time has been wasted.

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A lot of research has been done to realize auto hex-mesh generation in the past [3,4], and a series of algorithms have been given including grid-based methods, medial surface methods [5], advancing front techniques [6,7], etc.

Due to the robustness and effectiveness of grid-based method, it is widely used to generate meshes [8–10]. In order to generate adaptive mesh, two and three refinement templates were introduced [11]. Algorithms of 3-refinement is easy to carry out,therefore it was implemented first [12,13]. Because 2-refinement can produce less and smoother meshes than 3-refinement, many recent researches have focused on high-efficient octree-based methods so as to generate adaptive all-hex meshes [14,15]. Although robust, grid-based method tends to generate poor quality elements at the buffer zone between core mesh and boundary. Thus many quality improvement methods are proposed to refine the meshes generated including pillowing [16], geometric flow [17] and optimization-based smoothing [18].

Although mature and stable, all the aforementioned methods are weak in generating strictly structured hexahedral mesh. Mapping [19] and sweeping [20] can quickly generate structured hex-mesh. However, their scopes of application are immensely limited. As far as concerned, sub-mapping [21,22] is an ideal mapping method on account of the structured meshes that sub-mapping generates.

At the beginning of sub-mapping, sub-volumes topologically similar to hexahedron are obtained from the input quasi-polycube entity via real or virtual volume decomposition. These simpler sub-volumes can easily be meshed using well-known methods such as P.D.E.-based method, algebraic interpolation method, isoparametric method, etc. Meshes of all the sub-volumes comprise mesh of the entity. Although sub-mapping can generate structured hexahedral meshes validly, problems go with it.

Decomposing entity into sub-volumes topologically similar to hexahedron is a big challenge for sub-mapping. Clear descriptions of decomposition algorithms can rarely be seen from related literature. Actually, general decomposition algorithm can hardly be given due to the diversity of entity structures.

One decomposition algorithm may behave oppositely under different circumstances. Fig. 1a shows the results when decomposing using concave faces. It can be seen that decomposition succeeds when taking concave face A as a reference and fails when taking face B. In order to avoid decomposition failures, virtual decomposition algorithm was proposed [23]. After meshing all surfaces of the entity, an isosurface, a reference surface used to virtually decompose the entity, is constructed according to surface meshes' nodes at the same "elevation" in the computational domain. Virtual decomposition algorithm markedly improved the stability of decomposition algorithm, but problem still remains under especial situations. Fig. 1b demonstrates a flaw brought about by virtual decomposition. Isosurface, whose computational coordinate $I = 2$, is taken as a reference surface. The intersecting between the reference surface and the upper surface whose computational coordinate $I = 4$ causes failure of the virtual decomposition process.

Moreover, most entities for sub-mapping are similar but not strictly identical to polycubes. A number of algorithms have been proposed to obtain polycubes corresponding to original models. Su et al. [24] tessellated surfaces of the original model and utilized fuzzy logic method to adjust orientations of the tessellated faces in order to obtain polycube. Gregson et al. [25] made use of the mesh morphing technology which gradually transforms original entity into polycube. Although these methods can validly obtain polycube of original entity, subtle differences exist between polycube and original entity. Under certain circumstances, these differences may lead to unsatisfactory mesh quality.

A typical process of hex-mesh generation is demonstrated in Fig. 2. By use of certain morphing algorithm, original entity Fig. 2a is transformed into polycube Fig. 2b. Mesh Fig. 2c is fetched by meshing polycube Fig. 2b in the computational domain. Final mesh Fig. 2d is then obtained by transforming mesh Fig. 2c back to the original entity. However, it is because of polycube Fig. 2b having four more edges than original entity that abnormities occur in the final mesh Fig. 2d. Actually, having Fig. 2a structure similar to hexahedron, original entity Fig. 2a can simply be meshed into mesh Fig. 2e by mapping method. The minimum Jacobi value of mesh Fig. 2d is merely 0.38, which is greatly lower than 0.87 of mesh Fig. 2e. Each node of a hexahedron has its corresponding scaled Jacobian [26]. Jacobi value of mesh mentioned in this paper refers to the minimum scaled Jacobian values of nodes of all hexahedrons of the mesh. Aspect ratio of an element mentioned in this paper refers to ratio of lengths of the longest edge to the shortest edge in the element.

Ruiz-Gironés and Sarrate [27] attempted to construct a integer linear programming by using digraph corresponding to the original entity and result of classification of orientation of edges to obtain polycube. This method is referred to as LPM (Linear

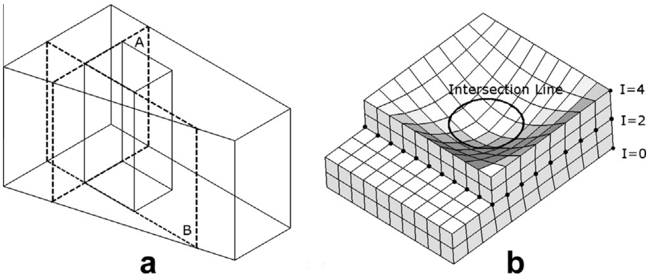

Fig. 1. Illustration of decomposition flaws.