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A discrete convolutional Hilbert transform with the consistent imaginary initial conditions for the time-domain analysis of five-layered viscoelastic sandwich beam

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ABSTRACT

A discrete convolutional Hilbert transform (DCHT) with the consistent imaginary initial conditions, together with the development of 2-node 8-DOF damped beam element, are presented for the reliable DOF-efficient time-domain analysis of five-layered viscoelastic sandwich beam. Motivated by the fact that the longitudinal displacements of three metallic layers can be replaced with the transverse shear strains of two viscoelastic core layers, a DOF-efficient damped beam element with the nodal DOFs composed of the deflection and rotation of beam and shear strains of two viscoelastic core layers is derived according to the virtual work principle and the compatibility relation. The standard Hilbert transform using Fourier and inverse Fourier transform of impulse signals produces the totally different results from the analytically derived ones near the end of time period, and the non-conjugate complex eigen values in a state-space formulation cause the unbounded growth in the time response of the damped structural dynamic system when a standard time integration scheme is used. To resolve these numerical problems, the imaginary external force is obtained by dividing the real external force into a finite number of rectangular impulses and by superposing Hilbert transforms of each rectangular impulse. And the time response of the damped sandwich beam subject to arbitrary external force is obtained by the convolution of time response to unit impulse. Meanwhile, the consistent imaginary initial conditions which can provide the bounded damped time response are numerically derived by splitting each decoupled complex second-order differential equation in the mode superposition approach into real and imaginary ones and by solving general solutions of each two split equations in the space-state formulation. The proposed method is validated through the numerical experiments composed of analytic and five-layered damped sandwich beam examples.

1. Introduction

The repression of the structural vibration and noise has been a great challenging subject in various engineering fields during several decades, because a dynamic system or its components with insufficient damping may, but frequently, lead to the dynamic instability and undesired noise. According to the very intensive continuing research efforts, a lot of useful

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passive and active devices for reducing the structural vibration have been introduced, and viscoelastic material among the damping materials used for such devices is widely used to dissipate the structural vibration energy. Viscoelastic layers inserted between the metallic layers exhibits the significantly high damping effect, called the constrained-layer damping [1–3], according to the high shear deformation of viscoelastic layers. The structural vibration of a constrained-layer sandwich beam is characterized by two distinct deformation modes, the oscillating flexural bending deformation of metallic faces and the alternating distortional shear deformation of viscoelastic layer. The transverse shear strain of the viscoelastic layers which is induced by the oscillating flexural bending of the metallic members produces the transverse shear stress with the phase lag. Thus, the oscillating vibratory energy of the sandwich structure is dissipated via the hysteretic loss of the viscoelastic layers [4,5].

The constrained-layer damping has been continuously and intensively studied since the late 1950s, and most of them were motivated by RKU (Ross–Kerwin–Ungar) theory of Ross et al. [1]. They laid down the basic mathematical framework for the viscoelastic constrained-layer sandwich beam and derived the effective, complex and flexural stiffness for the beam section. Since then, based on RKU theory, DiTaranto [2] and Mead and Markus [3] derived the six-order differential equations governing the natural frequencies, the associated composite loss factors and the forced vibration of three-constrained-layer damped beams by introducing the complex shear modulus. Thereafter, the extensive research efforts have been made by the subsequent investigators [6–8], in order to refine the earlier theories by including the additional damping effects due to the extensional/compressive deformations and the rotary inertia and to analyze the forced vibration of various multi-layered viscoelastic sandwich structures [4,9–12].

Sainsbury and Zhang [13] introduced a new more accurate and efficient Galerkin element with eight DOFs for the forced vibration analysis of unsymmetrical three-layer damped sandwich beam in which the displacement compatibility over the entire interfaces between the damping and elastic layers is taken into consideration. Trindade et al. [14] proposed an electrically coupled beam element with eight DOFs to handle the hybrid active–passive multilayer sandwich beam structures, where the frequency-dependence of the viscoelastic material is handled through the anelastic displacement fields (ADF) model. Galucio et al. [15] presented a finite element formulation and an 8-DOF damped sandwich beam for the transient dynamic analysis of sandwich beams with embedded viscoelastic material. Shorter [16] introduced a spectral finite element method using 1-D finite element mesh to efficiently compute the lower-order wave types and damping loss factors of a viscoelastic laminate. Plagianakos and Saravanos [17,18] presented an integrated high-order layerwise formulation and a 2-node damped beam element for predicting the damped free-vibration and thick composite sandwich beams, for which quadratic and cubic fields are added to the linear layerwise formulation in the kinematics of each discrete layer while maintaining displacement compatibility. Moreira et al. [19] developed a 4-node quadratic facet-shell finite element, based on a generalized layerwise formulation, to simulate multiple viscoelastic layer or multiple soft core sandwich plates.

More recently, Amichi and Atalla [20] introduced a damped beam element with eighteen DOFs for the forced vibration analysis of three-layer curved symmetric and asymmetric sandwich beams with a viscoelastic core based upon the discrete displacement approach. The in-plane and transverse displacements are interpolated with C^0 continuous linear and cubic polynomials respectively, and the rotational influence of the transversal shearing in the core. Ghinet and Atalla [21] introduced an analytical discrete laminate method to model thick composite laminate and sandwich plates and beams with linear viscoelastic damping layers, which can handle symmetric and asymmetric layouts of unlimited number of transversal incompressible layers. Assaf [22] extended his previous displacement-based FE formulation for three-layer damped sandwich plates to sandwich beams made up of cross-ply laminate faces with arbitrary number of 0° and 90° plies and a viscoelastic core. The formulation was based on a layerwise linear axial displacement through the beam thickness and a 2-node 8-DOF damped beam element was developed using Lagrange linear functions for the mean and relative axial displacements and Hermite cubic functions for the transverse displacement.

Most of previous works are concerned with the forced vibration analysis in the frequency domain and the development of efficient damped beam elements for layered damped sandwich beam structures. In case of the frequency-domain analysis, the external excitation force is expressed as harmonic force so that the imaginary part is automatically defined. But, in case of the time-domain analysis of damped sandwich beam structures, the external excitation force is real contrary to the complex-valued forced dynamic equation. In addition, a state-space formulation to find the time response of the damped dynamic system leads to two poles which are radial symmetry in the complex plane [23]. The radial symmetry of two poles implies that one pole is stable but the other is unstable such that the use of standard technique for solving the damped dynamic system will not result in a successful solution because the unstable pole will generate unbounded growth of the impulse response of the system.

In order to maintain the consistency in the complex-valued forced dynamic equation for the forced vibration of damped sandwich structures, the real-valued external impulse force is replaced with an analytic force function by generating the imaginary force signal using Hilbert transform [23–25]. However, the Hilbert transform of impulse force using discrete Fourier transform (FT) and inverse Fourier transform (IFT) may produce imaginary force signal different from one which is analytically derived from the definition of Hilbert transform. Therefore, there is no doubt that the analytic impulse force generated by discrete FT and IFT leads to incorrect time response of the damped dynamic system. Meanwhile, the time-reversal technique [26,27] was introduced to avoid the unbounded growth of the time response and widely used in acoustic and medical applications. In this technique, the time differential equation corresponding to the unstable pole is converted to one running backwards in time from zero to negative infinity, in which the initial conditions of the system should be imposed at negative infinity. In acoustic and medical applications where the time responses are obtained based on outdoor