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A hierarchical detection framework for computational contact mechanics

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ABSTRACT

A novel methodology consisting of three hierarchical levels is proposed for the detection phase of contact mechanics simulations. The top level of the hierarchy uses kinematic information from the objects involved in the simulation to determine approximate collision times. These instants then determine when the engine resumes operation for further detection. By using bounding volume hierarchies, the second level of detection precludes contact by computing simple exclusion tests on bounding volumes of increasing tightness. When contact cannot be ruled out by using simple tests, the final level of detection comes into effect by using thorough checks on finite element primitives. To that purpose, a robust optimization-based formulation that does not rely on orthogonal projections is outlined. The detection framework can be used to predict the exact collision time among finite element discretizations. The performance of the proposed methodology is investigated with a set of examples.

1. Introduction

The science of *contact mechanics* originated in the early 1880s with the work of Hertz [1], who solved the problem of two elastic bodies with convex boundaries in contact. The advent of computers, and the development of the finite element method (FEM) for the simulation of a wide range of engineering problems, gave rise to the field of *computational contact mechanics*. The latter aims at providing the computational means for the prediction of the physical processes that occur when simulated solid bodies come into contact with one another. Diverging from a purely theoretical framework, the latter has found applications in a wide range of areas, including the analysis of crashworthiness [2], metal forming [3,4], shell structures [5], and impact and penetration [6]. The implementation of a contact mechanics algorithm can be roughly subdivided into two major components: (i) *contact detection*, and (ii) *contact resolution*. This article discusses the first major component, as the detection phase can encompass a major portion of the CPU time in the simulation of contact, specially with explicit integration time stepping algorithms [7]. As an example, Attaway et al. [8] allot 30% to 60% of the computational time to the contact detection phase on the Cray Y-MP vector supercomputer.

In computer science, *collision detection* describes the procedure by which the intersection between at least two objects is determined. Collision detection has found applications in computer graphics visualization [9,10], virtual reality [11], computer-aided design (CAD) [12,13], game development [14], and robotics [15–17]. Collision detection checks are typically carried out following a space decomposition or on hierarchies of bounding volumes, or even a combination thereof. The objective of the former is to subdivide the search space so that the number of collision checks is minimized. Choices for space

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decomposition can consist of tree data structures, including binary space partition (BSP) trees [18,19], *k*-d trees [20,21,11,22], octrees [23–25], R*-trees [26], tetrahedral meshing [11], and grids [27]. Bounding volumes typically chosen for building hierarchies include bounding spheres (BSs) [9], axis-aligned bounding boxes (AABBs) [27], discrete orientation polytopes (*k*-DOPs) [10], oriented bounding boxes (OBBs) [28], and convex hulls [29]. For survey articles on collision detection, the reader is referred to [30–32] and the references therein. Also, a thorough treatment on collision detection can be found in the book by Ericson [33].

The *contact resolution* is the phase of the simulation that determines the response of colliding objects. Under the terminology of *collision response*, the computer science literature lists a substantial body of work, starting from the early works on the response between rigid bodies [24,34–36], to the study of deformable bodies [37–39], cloth animation [40–44], fragmentation [45,46] and even hair assemblies [47,48]. Even though most of the works on collision response in the computer graphics literature are simplistic in regards to the physics, some newer articles address the response in a more rigorous manner, e.g., by including adhesive contact [49] and friction [50,51,47,48,44]. The finite element method has also been used for more accurate representations of the response [39,44]. Note that this is by no means an exhaustive list.

This article emphasizes on the contact detection phase, and as expressed above, there is a wealth of knowledge from the computer science community that could successfully be applied to the field of computational contact mechanics. We will focus mainly on collision checks on bounding volume hierarchies, as this procedure can be combined with any aforementioned space decomposition technique. Gottschalk et al. [28] employed hierarchies of OBBs and showed that they outperform the use of BS and AABB hierarchies in situations of close proximity. Klosowski et al. [10] studied the use of bounding volume trees based on *k*-DOPs and showed that their algorithm can perform the collision detection of hundreds of thousands of polygons at interactive rates. Otaduy and Lin [52] introduced the idea of *contact levels of detail* (CLOD), by which they use a dual hierarchy of the model for accelerating the collision detection phase. One hierarchy is a multiresolution representation of the model, while the other one uses bounding volumes. In a recent publication, Larsson and Akenine-Möller [53] combine the BSs, OBBs, and *k*-DOPs into *slab cut balls*, which are hybrid bounding volumes that outperform both BS and OBB hierarchies.

In the computational contact mechanics literature there is only a handful of works that adopt some of this knowledge. Grids are the space decomposition methodology of choice [5,54]. The straightforward implementation of this technique, that is also referred to as *bucket search*, may be the culprit for its prevailing adoption among computational mechanicians. A global contact search methodology based on the sorting of slave nodes is also described by Heinstein et al. [54]. Yang and Laursen [7] have recently implemented the detection based on hierarchies of *k*-DOPs in the context of contact simulations based on mortar formulations. Nevertheless, it is the view of the authors that there is a considerable time lag between the two fields of study. This manuscript is the authors' attempt on using some of the newer knowledge gathered by the computer science community and apply it to the field of computational contact mechanics.

In this article we propose a novel three-level detection phase for contact mechanics. At the upper level, a *trajectory based* approach is used to compute the approximate time the detection engine initiates to operate. In the literature, a fixed time step is normally used to perform the detection, and collisions can be missed depending on the choice of this parameter. By determining an approximate time of a collision, the proposed approach is robust as it guarantees the detection of all collisions. A second level of detection consists of carrying out intersection tests on hierarchies of bounding volumes. Initially, the simplest bounding volumes are used, and adaptivity is applied to the bounding volumes as more precise detection tests are needed. The final stage of the detection phase consists of thorough checks for actual contact, by computing distances from nodes to finite elements using an optimization-based approach. An appropriate contact detection engine has to be both efficient and accurate [15], for it is important that precise contact detections are determined with the least amount of time. In the proposed scheme, the upper two layers are concerned with efficiency, while the last layer deals with accuracy. The proposed strategy can prove invaluable in speeding up this computationally-intensive stage of physics simulations involving contact.

The article is organized as follows: the description of the contact problem, together with mathematical definitions of bounding volumes and their corresponding hierarchies, is provided in Section 2. Section 3 outlines the proposed hierarchical approach to contact detection. Implementation details of the framework are given in Section 4. Finally, Section 5 presents some examples of the use of the proposed methodology.

2. Problem description

Let $\mathbb{R}^d$ represent the d-dimensional Euclidean space, where a coordinate is represented by $\boldsymbol{x} = x_i\boldsymbol{e}_i$, and $\boldsymbol{e}_i$ is a chosen orthogonal basis. The space is equipped with inner product $\langle\cdot,\cdot\rangle$ that induces the norm $\|\boldsymbol{x}\| = \sqrt{\langle\boldsymbol{x},\boldsymbol{x}\rangle}$. Let $\Omega_i \subset \mathbb{R}^d$ be the mathematical representation of the ith object involved in the simulation, as shown in Fig. 1. The boundary of the domain $\partial\Omega_i := \overline{\Omega}_i \setminus \Omega_i$, with unit normal $\boldsymbol{n}_i$, can be decomposed into mutually exclusive regions $\partial\Omega_i^u \cup \partial\Omega_i^t \cup \partial\Omega_i^c$. The regions $\partial\Omega_i^u$ and $\partial\Omega_i^t$ correspond to those where Dirichlet and Neumann boundary conditions are prescribed, respectively. The contact constraints are enforced in $\partial\Omega_i^c$, that depending on the problem, may be equal to $\emptyset$ most of the simulation time.

For each individual domain Ω_i, we are interested in obtaining its displacement $\boldsymbol{u}(\boldsymbol{x},t) : \Omega_i \times T \to \mathbb{R}^d$, and corresponding velocity $\dot{\boldsymbol{u}}$ and acceleration $\ddot{\boldsymbol{u}}$ fields, for a time period $T \subset \mathbb{R}$. The initial elasto-dynamics boundary value problem is stated as: given the body density $\rho_i : \Omega_i \to \mathbb{R}$, body force $\boldsymbol{b}_i : \Omega_i \times T \to \mathbb{R}^d$, initial displacement and velocity $\boldsymbol{u}_i^0, \dot{\boldsymbol{u}}_i^0 : \Omega_i \to \mathbb{R}^d$,