Comput. Methods Appl. Mech. Engrg. 268 (2014) 632–655

Contents lists available at ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Stress-related topology optimization of continuum structures involving multi-phase materials

Xu Guo *, Weisheng Zhang, Wenliang Zhong

State Key Laboratory of Structural Analysis for Industrial Equipment, Department of Engineering Mechanics, Dalian University of Technology, Dalian 116023, PR China

ARTICLE INFO

Article history:
Received 6 July 2013
Received in revised form 29 August 2013
Accepted 4 October 2013
Available online 26 October 2013

Keywords:
Topology optimization
Stress constraints
Level set
Shape derivative
Extend Finite Element (X-FEM)

ABSTRACT

Considering stress constraints in structural topology optimization is very important from both theoretical and application perspectives. Most of the corresponding studies in literature, however, are only focused on the problems involving single phase homogenous materials. In the present paper, stress-constrained topology optimization of continuum structures involving multi-phase heterogeneous materials is investigated. A level-set based variational consistent solution framework is developed. Numerical examples are also presented and discussed to illustrate the effectiveness of the proposed approach.

1. Introduction

Since the pioneering work of Bendsoe and Kikuchi [1], many efforts have been devoted to the optimal topology design of structures. Nowadays, topology optimization has reached a certain level of maturity and become a rather well-established research field. Recent years also witnessed numerous successful applications of topology optimization methods in real world industrial applications. It is not intended to give a state-of-the-art review here. For a more detailed description of the theories and methods developed in recent years, we refer the readers to [2–4] and the references therein.

Traditional researches on topology optimizations are mainly focused on stiffness-oriented problems, where structural compliance or displacement is often selected as concerned objective or constraint functional. The strength constraints, however, are very important for the design of engineering structures and products. Without considering strength constraints appropriately, the reliability of a design cannot be guaranteed since too high local stress will lead to the fracture, damage, fatigue or even the failure of structures. In view of this, there is a growing interest in the study of topology optimization problems considering stress-related objective or constraint functionals.

Topology optimization problem of continuum structures with point-wise stress constraints was first addressed by Duysinx and Bendsoe [5]. In their work, the so-called micromechanics consistent scheme was used for stiffness and allowable stress interpolations under SIMP (Solid Isotropic Material with Penalization) framework. Moreover, the so-called ε-relaxed approach [6] was applied to deal with the singularity phenomenon caused by local stress constraints which was first analyzed correctly in [7]. Pereira et al. [8] also studied the topology optimization with material failure constraints by

* Corresponding author. Tel.: +86 411 84707807.
E-mail address: guoxu@dlut.edu.cn (X. Guo).

0045-7825/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cma.2013.10.003

employing the ε-relaxed approach. Their study showed that optimal designs obtained under material failure constraints may be quite different from that obtained under maximum stiffness criteria. Bruggi et al. [9,10] proposed an alternative relaxation approach (i.e., *qp*-relaxation approach) for stress-constrained topology optimization to circumvent the difficulties caused by singularity phenomenon. Recently, Le et al. [11] proposed a new approach for stress-constrained topology optimization problems. In their approach, a global/regional stress measure combined with an adaptive normalization technique was suggested to control the local stress level in a computational efficient way. Holmberg et al. [12] proposed an algorithm for stress-constrained topology optimization problems with use of a clustering technique, where the stresses at several stress evaluation points are clustered into groups to alleviate the computational cost arising from the local stress constraints.

Besides the above SIMP-based approaches, level set-based method has also been used to deal with the stress-related topology optimization problems. Allaire and Jouve [13] considered the stress minimization topology design problems by employing a global stress measure in an integral form in the level-set framework. In [14], the authors discussed the challenging issues for stress-related topology optimization problems and demonstrated how to resolve these issues in a level-set based framework. They also pointed out that when local stress constraints are considered, global structural performance measures should be included into the problem formulation in order to regularize the solution process. Zhang et al. [15] proposed a new global stress measure enhanced by stress gradient information to control the local stress level in level-set based framework. Numerical examples demonstrated that with use of the suggested global stress measure, local stress level can be controlled successfully without introducing any artificial parameters. Later on, Wang and Li [16] introduced the concept of shape equilibrium constraint for stress-constrained structural optimization. Level-set framework was also adopted for problem formulation. Xia et al. [17] also proposed a global stress measure with multiple parameters for stress-constrained structural optimization under level set description. In order to increase the accuracy of stress computation, adaptive finite element re-meshing has been carried out instead of using the artificial weak material to mimic voids, which is a common practice in conventional treatment of topology optimization problems. Furthermore, Amstutz and Novotny [18] and Suresh [19] illustrated how to deal with stress-related topology optimization problems effectively by combining the level-set description and topological derivative information.

Although remarkable achievements have been made for stress-related topology optimization problems, it is worth noting that most of the studies mentioned above are devoted to the problems with single-phase homogenous materials. In modern engineering applications, however, the consideration of structures made in multi-phase materials is unavoidable and has become increasingly concerned. This is due to the fact that outstanding structural performance (e.g., hard and ductile, ultra-light and tough) can be achieved by combining different single-phase materials appropriately, and therefore the resulting structures can be employed to fulfill functions that cannot be accomplished well by individual single-phase materials. Therefore how to design structures composed of multi-phase materials in a rational and efficient way has received ever-increasing attention in recent years.

From theoretical point of view, designing structures composed of multi-phase materials can also be formulated as a topology optimization problem, where optimal multi-phase material distribution is sought for. Compared to black-and-white topology optimization where only single-phase material is considered, however, topology optimization involving multi-phase materials is more difficult to deal with since the typical challenges in topology optimization, such as topology description parameterization, boundary blurring control and accurate stress computation, are more severe in the latter case. Furthermore, it is very difficult or even impossible to guess the optimal topology of multi-phase materials based on engineering intuitions. Existing works on topology optimization with multi-phase materials can be found in [20–26]. But none of them discussed the case where stress constraints are concerned.

Encouraged by the results obtained in [14,15], in the present paper, we intend to discuss how to carry out stress-related topology optimization with multi-phase materials under level set-based computational framework. In particular, we will demonstrate how to derive the shape derivatives, which is an important ingredient of level set-based numerical optimization, under Eulerian description in a variational consistent and correct way when multi-phase material are involved. The rest of the paper is organized as follows. In Section 2, the statement of the considered topology optimization problem and its mathematical formulation is presented. A level-set based solution framework is then described in detail in Section 3. Section 4 is devoted to the discussion of the corresponding numerical solution aspects. In particular, we also show in this section how to perform shape sensitivity analysis based on Eulerian description. The effectiveness of the proposed approach is demonstrated through several numerical examples in Section 5. Finally some concluding remarks are provided in Section 6.

2. Problem statement and its mathematical formulation

In the present work, topology optimization of continuum structures under stress constraints is considered. In contrast to the corresponding studies in [14,15], emphasis is placed here on dealing with the case of multi-phase materials. To this end, he mathematical formulation suggested in [15] is adopted in the present work and it can be written in a general form as follows: