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A gradient-enhanced large-deformation continuum damage model for fibre-reinforced materials

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ABSTRACT

A non-local gradient-based damage formulation within a geometrically non-linear setting is presented. The hyperelastic constitutive response at local material point level is governed by a strain energy which is additively composed of an isotropic matrix and of an anisotropic fibre-reinforced material, respectively. The inelastic constitutive response is governed by a scalar [1–d]-type damage formulation, where only the anisotropic elastic part is assumed to be affected by the damage. Following the concept in Dimitrijević and Hackl [28], the local free energy function is enhanced by a gradient-term. This term essentially contains the gradient of the non-local damage variable which, itself, is introduced as an additional independent variable. In order to guarantee the equivalence between the local and non-local damage variable, a penalisation term is incorporated within the free energy function. Based on the principle of minimum total potential energy, a coupled system of Euler–Lagrange equations, i.e., the balance of linear momentum and the balance of the non-local damage field, is obtained and solved in weak form. The resulting coupled, highly non-linear system of equations is symmetric and can conveniently be solved by a standard incremental-iterative Newton–Raphson-type solution scheme. Several three-dimensional displacement- and force-driven boundary value problems—partially motivated by biomechanical application—highlight the mesh-objective characteristics and constitutive properties of the model and illustratively underline the capabilities of the formulation proposed.

1. Introduction

The understanding and physical description of damage and failure effects in materials presents a major challenge in various engineering-related disciplines. Failure processes in ductile steels, for instance, are characterised by a complex interaction of plastic deformation and damage effects. These phenomena originate from the motion and accumulation of dislocations which interact and result in micro-cracks and micro-voids and finally lead to a necking-like reduction of effective area and possibly to ultimate failure of the material. Damage processes in concrete are typically initiated at the interface between stiff grains and the ambient cement matrix, result in a coalescence and propagation of micro-cracks and finally lead to a global stiffness loss accompanied by localised degradation patterns. To give a particular example with regard to the present contribution, damage processes in anisotropic soft biological tissues are closely related to the progressive failure of fibres embedded in the ambient bulk or matrix material. An exemplary qualitative simulation result based on the constitutive

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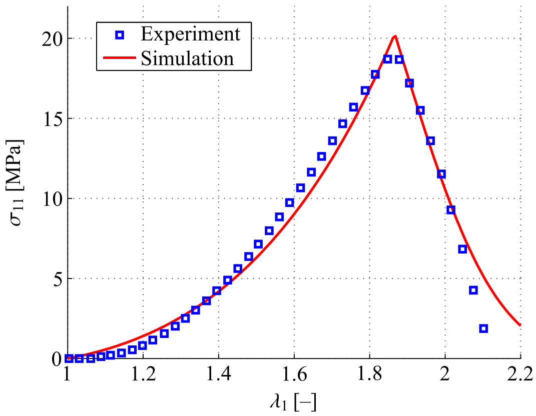

Fig. 1. Qualitative simulation based on the constitutive model proposed in Section 3 and experimental stress-stretch data of a fascia tissue specimen under uniaxial isochoric tension, cf. Martins et al. [58]. Material parameters are chosen as $\mu_e = 0.01$ kPa, $\kappa_e = 499.0$ kPa, $k_1 = 0.55$ kPa, $k_2 = 0.01$, $\kappa_d = 3.5$ kPa, $\eta_d = 0.6$ kPa^{-1}.

model subsequently proposed in this contribution, together with experimental stress-stretch data from Martins et al. [58], is provided in Fig. 1. The results refer to the mechanical response of a fascia tissue specimen under uniaxial isochoric tension. Arteries, for instance, can be considered as a composite of an isotropic ground substance of elastin fibres and a highly anisotropic network of cross-linked collagen fibrils. Mechanical loading beyond the physiological loading range, e.g., caused by a surgical intervention such as balloon angioplasty, can significantly reduce the elastic properties of the artery. Physically related to Mullins-type effects, these phenomena can be attributed to the continuous degradation of particular collagen fibres and to the corresponding overstretch of neighbouring cross-links leading to pronounced softening.

Based on the classical work by Kachanov et al. [43] who interpreted the damage effect as a consequence of an area reduction of the stress-bearing region, material degradation can be modelled by means of standard continuum damage formulations, i.e., in a local sense. Up to now, a large variety of models exist where we refer the reader to classic monographs and textbooks as, e.g., Kachanov [43], Kachanov [44], Krajcinovic and Lemaitre [46], Lemaitre and Chaboche [51], Lemaitre [50] or Krajcinovic [45], to name only a few. However, the assumption of a purely *local* continuum damage may, as a major drawback, imply a loss of well-posedness, such as loss of ellipticity, of the underlying boundary value problem. With regard to related numerical methods such as the finite element method, this results in a significant mesh-dependence of the solutions, in other words in a vanishing localised damage zone upon mesh refinement, and hence physically meaningless—or at least questionable—simulations.

In order to regularise the problem, and thereby to circumvent the aforementioned deficiencies, several approaches have been proposed in the literature as, for instance, viscous regularisation, or the concept of generalised *non-local* continua such as micromorphic continua, see the monograph on non-local continuum field theories by Eringen et al. [31], the articles by Aifantis et al. [7,8] or the contributions in Eringen [30] and Rogula [81]. Here, intrinsic or rather internal length scales are introduced into the continuum formulation. A non-local continuum formulation can generally be established by either introducing an integral- or a gradient-type equation.

Non-local *integral* models are inherently associated with a global averaging procedure which complicates the linearisation of the equations and from the computational point of view, are far more expensive than related gradient-type models. With regard to continuum damage formulations, non-local integral models are advocated by Pijaudier-Cabot and Bažant [73] and Bažant [16] and extensively studied by Jirásek [42] or Bažant and Ožbolt [18] and Bažant and Di Luzio [17], the latter referring to microplane models.

As a convenient alternative, Lasry and Belytschko [49], Mühlhaus and Aifantis [64], Polizzotto et al. [74] suggested non-local *gradient* models. Here, the non-locality is incorporated by means of an additional gradient-based Euler–Lagrange equation, to be fulfilled in a weak sense, accompanied by a non-local quantity taking the interpretation of an additional independent variable. Non-local gradient-based continuum damage formulations were first discussed in de Vree et al. [94], Peerlings et al. [71] and de Borst and Pamin [21]; see also the comparison by Pamin [69] or the coupled damage-plasticity approach by Svedberg and Runesson [91].

A large variety of gradient-extended damage formulations exists for the geometrical *linear* case, see e.g., the contributions by Kuhl and Ramm [47] and Kuhl et al. [48] for anisotropic gradient damage or the article by Liebe et al. [55] for isotropic gradient damage. However, there is a comparatively small number of contributions for the geometrical *non-linear* case. The article by Steinmann [90] can be considered as a starting point wherein the non-local strain energy density is introduced as an additional primary variable. This approach was used in Liebe and Steinmann [54], and similarly with application to softening-plasticity in Liebe et al. [53], and compared to an alternative model which takes the damage field as an independent variable. Both approaches used a global active-set-search to account for the Kuhn–Tucker conditions. Recently, Wcisło et al. [97] proposed a gradient-enhanced large-strain damage-plasticity model where gradient averaging is applied to the deformation measure which determines the damage evolution. Following the concept by Frémond and Nedjar [33], the article by Nedjar [68] considered a damage-related formalism based on the principle of virtual power where the power of the internal