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A consistently coupled isogeometric–meshfree method

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ABSTRACT

A consistently coupled isogeometric–meshfree method is presented. This method takes advantage of the geometry exactness of isogeometric analysis and the refinement flexibility of meshfree method. The coupling of isogeometric approximation and meshfree approximation is based upon the reproducing or consistency conditions which are crucial for the coupled method to achieve the expected optimal convergence rates. It is shown that unlike the reproducing kernel meshfree shape functions which satisfy the reproducing conditions with the nodal points as the reproducing locations, the monomial reproducing points for different orders of B-spline basis functions in isogeometric analysis are different and consequently a rational method is proposed to compute these reproducing points. Both theoretical proof and computational justification of the reproducing conditions for B-spline basis functions are given. Subsequently within the framework of reproducing conditions, a mixed reproducing point vector is proposed to ensure arbitrary order monomial reproducibility for both B-spline basis functions and reproducing kernel meshfree shape functions, which leads to a consistently coupled approximation with smoothing transition between B-spline basis functions and reproducing kernel meshfree shape functions. Consequently a coupled isogeometric–meshfree method is established with the Galerkin formulation. The effectiveness of the proposed coupled isogeometric–meshfree method is demonstrated through a series of benchmark numerical examples.

1. Introduction

The isogeometric analysis initiated by Hughes et al. [1] provides an elegant and seamless integration of the computer aided design and the finite element analysis. In this method the non-uniform rational B-splines (NURBS) for exact geometry representation are directly adopted for the finite element analysis and consequently the model discretization error is completely eliminated. Arbitrary high order smoothing approximation with geometry exactness can be readily realized in the context of isogeometric analysis [1,2]. The properties of approximation smoothness and geometry exactness yields superior solution accuracy for the isogeometric analysis compared with the conventional C^0 finite element analysis, which is thoroughly demonstrated by fluid mechanics problems [3–7], fluid–structure interactions [8–10], structural dynamics problems [11–13], contact problems [14,15], design optimization problems [16–18], etc. Meanwhile, the smoothing approximation characteristic makes the isogeometric analysis very suitable for the high order problems such as phase field modeling, gradient-based elasticity and damage analysis [19–21], and beam, plate and shell problems[22–27]. The isogeometric analysis was also extended to enhance the performance of finite element [28,29], boundary element [30,31], and meshfree methods [32]. More comprehensive discussion about the developments of isogeometric analysis can be found in [2,33].

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The NURBS-based isogeometric analysis inherits geometric exactness during the mesh refinement process, nonetheless multidimensional NURBS basis functions and related mesh refinement rely on the tensor product operation which poses severe difficulty for the flexible local mesh refinement. An effective approach to overcome this difficulty is to employ the T-splines for isogeometric analysis [34–38], in which the linear dependence problem needs be properly resolved [39]. Alternatively, a quite straightforward local model refinement can be achieved in the meshfree setting and very recently Rosolen and Arroyo [40] proposed a blending of isogeometric analysis and local maximum entropy meshfree approximants. In this approach, the isogeometric and local maximum entropy meshfree basis functions are coupled through minimizing the local entropy function under linear reproducibility constraints. Iterations are required for solving the minimization problem to obtain the basis functions. However, extension to higher order reproducing conditions for this coupling method with local maximum entropy meshfree basis functions is not trivial [41,42]. Moreover, it is shown in the present work that the exact reproducing points for B-spline basis functions generally are not the knots or control points.

In this study, a consistently coupled isogeometric–meshfree method with arbitrary order monomial reproducibility is proposed to take advantage of the isogeometric analysis and meshfree method, i.e., the geometry exactness, refinement flexibility, and the smoothing approximation shared by the two methods [43]. The meshfree approximation discussed herein particularly refers to the reproducing kernel approximation [44–50], which is also equivalent to the moving least square approximation [51–53] when monomial basis functions are utilized. The reproducing conditions have been employed by Liu et al. [54] and Huerta and Fernandez-Mendez [55,56] to enrich and couple the finite element and meshfree methods. Chen et al. [57] developed an interpolatory meshfree approximation with reproducing conditions-based nodal enrichment. A Lagrangian interface-enriched meshfree method was also presented by Wang et al. [58] for homogenization analysis based on the reproducing conditions. As for the reproducing kernel approximation, the shape functions are constructed based on the reproducing or consistency conditions and thus any order basis functions in the shape functions can be exactly reproduced with the meshfree nodes being the reproducing points [44–50]. On the other hand, to the authors' knowledge the reproducing conditions have not been discussed for B-spline basis functions used in isogeometric analysis. It turns out that the reproducing conditions with proper reproducing points are critical for the coupled isogeometric–meshfree method to achieve the optimal convergence rates.

In order to consistently couple the isogeometric analysis and meshfree method within the framework of reproducing or consistency conditions, the reproducing conditions for isogeometric basis functions, i.e., the B-spline basis functions are addressed in detail. It is shown that the knots or control points are not the reproducing points for B-spline basis functions and the reproducing points for different orders of B-spline basis functions are different. A rational method with theoretical proof is then presented to compute the reproducing points for B-spline basis functions. The reproducing conditions of B-spline basis functions are further verified through several numerical examples. Subsequently based on the reproducing conditions the coupling of isogeometric basis functions and meshfree shape functions is established. In the coupled region the coupled shape functions rely on a mixed reproducing point vector which enables the monomial reproducibility and smooth transition between the isogeometric and meshfree approximants. Thereafter a coupled isogeometric–meshfree method is established using the proposed coupling methodology and the standard Galerkin solution procedure. The effectiveness of the present method is systematically investigated through a series of benchmark examples.

An outline of the remainder of this paper is as follows. In Section 2, the meshfree and isogeometric approximations are summarized. Section 3 presents the reproducing conditions for B-spline basis functions, where the formula for computing the reproducing points of different order basis functions are detailedly discussed with computational verification. The coupling of isogeometric and meshfree approximations within the framework of reproducing conditions is introduced in Section 4. Thereafter several numerical examples are presented in Section 5 to demonstrate that optimal convergence can be achieved by the present coupled isogeometric–meshfree method. Finally conclusions are drawn in Section 6.

2. Meshfree and isogeometric approximations

2.1. Meshfree approximation

The parametric domain is used in this study to consistently couple the isogeometric and meshfree approximations. In meshfree approximation, the problem parametric domain Ω_ξ is discretized by a set of particles $\{\xi_A\}_{A=1}^{NP}$. The interaction between a particle $\xi_A = (\xi_A, \eta_A)$ and a field point $\xi = (\xi, \eta)$ is determined by the kernel function $\phi_a(\xi_A - \xi)$ associated with ξ_A. The influence domain of $\phi_a(\xi_A - \xi)$ is often denoted as $supp(\xi_A)$ such that $\Omega_\xi \subset \bigcup_{A=1}^{NP} supp(\xi_A)$, "$a$" is used to measure the size of $supp(\xi_A)$. The reproducing kernel (RK) meshfree approximant of a generic field variable, $u(\xi)$, commonly written as $u^h(\xi)$, takes the following form:

$$u^h(\xi) = \sum_{A=1}^{NP} \Psi_A(\xi) d_A \tag{1}$$

where $\Psi_A(\xi)$ is the meshfree shape function and d_A is a coefficient associated with node A. According to the reproducing kernel theory [44–50], $\Psi_A(\xi)$ can be expressed as:

$$\Psi_A(\xi) = \boldsymbol{p}^T(\xi_A)\boldsymbol{a}(\xi)\phi_a(\xi_A - \xi) \tag{2}$$