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Geometric modeling, isogeometric analysis and the finite cell method

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ABSTRACT

The advent of isogeometric analysis (IGA) using the same basis functions for design and analysis constitutes a milestone in the unification of geometric modeling and numerical simulation. However, an important class of geometric models based on the CSG (Constructive Solid Geometry) concept such as trimmed NURBS surfaces do not fully support the isogeometric paradigm, since basis functions do not explicitly represent the boundary. The finite cell method (FCM) is a high-order fictitious domain method, which offers simple meshing of potentially complex domains into a structured grid of cuboid cells, while still achieving exponential rates of convergence for smooth problems. In the present paper, we first discuss the possibility to directly couple the finite cell method to CSG, without any necessity for meshing the three-dimensional domain, and then explore a combination of the best of the two approaches IGA and FCM, closely following ideas of the recently introduced shell FCM. The resulting *finite cell extension to isogeometric analysis* achieves a truly straightforward transfer of a trimmed NURBS surface into an analysis suitable NURBS basis, while benefiting from the favorable properties of the high-order and high-continuity basis functions. Accuracy and efficiency of the new approach are demonstrated by a numerical benchmark, and its versatility is outlined by the analysis of different trimmed design variants of a brake disk.

1. Introduction

In recent years, the question of merging geometric modeling with numerical simulation has gained tremendous interest in the scientific as well as in the application oriented community. For industrial applications the relevance is obvious, as more than 80% (see [1]) of the overall cost of engineering design is typically devoted to the *transfer* of a geometry oriented design model to a finite element based computational model, whereas only a very small percentage is spent on the numerical computation itself. Both the design oriented as well as the computational oriented community have accordingly expressed a strong desire for tools which enable a smooth transition from one realm to the other. In the scientific world the importance of integrating modeling and computation has likewise been the subject of discussion for decades, although it has essentially had just a limited influence on engineering practice. It was the recent advent of the isogeometric analysis (IGA) that initiated a new wave of research and, within just a few years, introduced a vast variety of results, opening a new view on future concepts for design and simulation. The basic idea is as easy as it is convincing: Instead of using classical finite element basis functions living on a triangulated geometry, which is only a (more or less accurate) approximation of the geometric design model and therefore, needs be *derived* from the original model, IGA uses exactly the *same* shape functions for both computation and geometry description. Not only is the exact geometric representation a huge advantage, but the typically applied type of functions makes IGA superior to classical FEM in many aspects. Usually, B-splines or NURBS (see, e.g. [2–4]) are used, which are typically higher order approximates and allow for mesh refinement on the identical geometry as well as an increase in the degree of the underlying polynomial functions. Therefore, many of the desirable properties of the *p*-version of the finite element method (cf. [5–7]), such as robustness and excellent approximation properties, are inherited.

By nature, IGA is closely related to Boundary Representation (B-Rep) models, where a body is described by the shape of its surfaces and edge curves and where the topology consists of points, edges, loops, faces and lumps. See e.g. [8,9] for a detailed definition of the geometry and topology of B-Rep models which are widely used in current CAD systems. Whereas spline surfaces and curves generally provide a very powerful tool for describing a geometric shape, problems arise in the IGA, when e.g. trimmed patches are used to represent a body. A trimmed spline patch is a part of a spline surface or body, which is bounded by lower dimensional entities, i.e. in the case of a trimmed surface by one or more loops consisting themselves of curves on the surface. Several approaches to extend the IGA to trimmed surfaces have been suggested over the past years [10]. These range from a suitable re-parametrization

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(see [4]) to the generalization of NURBS-based IGA to T-spline based analysis [11], which also allows for a local refinement of the 'grid'. T-splines thus promise to restore one of the major advantages of classical finite elements, i.e. the possibility to use non-uniform meshes.

Looking at current CAD-systems on the whole, B-Rep models are by no means the only form of representation used for geometry. Constructive Solid Geometry (CSG) models (cf. [12]) are in many cases much more suitable for the design *process* than B-Rep models, as they make it possible to build up a structure in an intuitive and efficient way out of simpler sub-structures and generic geometric operations. Often used in conjunction with B-Rep descriptions, CSG-models are also one important basis for *parametric* [13] or *feature based* [14] design, where geometric entities are placed into relations and under constraints, and where a body is only defined implicitly by storing its entire construction process. There is virtually *never just one single* process for assembling a complex body, but the many different ways of construction lead to various models, some of which are better suited to isogeometric analysis than others. A designer may construct one and the same structure in a way which immediately fits IGA, or he may use a definition which is far removed from a straightforward transfer to analysis. Therefore, the simulation community often calls for an *analysis aware* design (cf. [15]) to facilitate the transition between the two worlds. In many cases it is not possible to fulfill this request as a typical designer is not 'aware' of the numerical simulation's needs and very often the design process itself only calls for CSG operations at a later stage of design, which - despite their fundamental simplicity - change the topology of a structure and accordingly the layout for suitable NURBS patches completely. 'Drilling a hole' in a structure would be one example for such an operation, where needs for an efficient design process and a suitable IGA-model are far apart. Whilst IGA is ideal for representing the boundary of a body by its 'mesh' it creates substantial problems for a straightforward transition from CSG models.

A new advancement in IGA technology has recently been introduced ([11,16]) where T-Splines are used to describe the geometry and solution fields, rather than of NURBS. They display a much larger flexibility from the point of view of local refinement, independence of the design process and (late) topological changes to a structure and therefore, overcome many of the aforementioned problems of NURBS-based IGA. Like the original IGA approach, they are still closely related to geometric boundary representation models rather than to Constructive Solid Geometry.

Exactly the opposite is true for *Embedded* or *Fictitious Domain Methods* (cf. [17–22]), which completely set aside the representation of the domain's boundary in a computational mesh and therefore, incur considerable effort to regain control over the precise shape of a structure. Yet these approaches can be closely linked to CSG-models, as will be shown in this paper. These approaches, also known as *Immersed Boundary Methods* [23,24], embed the domain of computation Ω into an extended domain Ω_e, typically with a geometrically simple shape, which can easily be discretized in a structured mesh or even a Cartesian grid for computation. This mesh or grid does not necessarily follow the boundary of the original domain Ω. An overview of pertinent literature is given, for example, in [25–28]. The recently proposed finite cell method (FCM) [29,30] uses basic concepts of fictitious domain approaches and extends them to high order Ansatz spaces familiar from the p-version of the finite element method. One core feature of this method is its capability to maintain high-order convergence rates and high accuracy although the geometry is only represented implicitly. Despite this astonishing feature, embedded domain approaches for high order finite element methods are still rarely found in literature. A recent overview of embedded domain methods in general is given e.g. in [31], including some references to higher-order approaches and focusing on methods which recover the original domain at integration level. A multi-level set technique to generate signed distance functions is advocated, which is then used to recover the original domain on the integration level. Another approach utilizes the Kantorovich method [32] in the finite element context. This method is implemented by *scan&solve* [33,34] and appears similar to the finite cell method at first glance. However, the basic idea here is to recover the solution in the original domain by multiplying the high-order shape functions with a function measuring the distance to the boundary of the structure. High-order embedded domain methods have also been reported in the context of the spectral element method [35], the extended finite element method [36] and interface problems [20].

The FCM has been investigated for linear elasticity in 2D and 3D [30,29,37], for topology optimization problems [38,39] and geometrically nonlinear problems [40–42]. A very fast implementation using pre-integrated stiffness matrices has been employed for interactive 3D-simulation in a computational steering system [43,44]. The finite cell method proves to have significant advantages over classical finite element methods or low order fictitious domain approaches in all these cases. Furthermore, adaptive schemes with hierarchical spline base functions have been developed [45], which feature certain connections to the isogeometric analysis. In [46] an extension of the FCM to shell-like solid structures using mapping concepts similar to the IGA has been proposed.

The particular advantages and disadvantages of IGA and FCM, i.e. their close relationship to BREP and CSG models respectively, call for an attempt to combine the best of both approaches. The goal of this paper is to show how the finite cell approach can support trimmed patch isogeometric analysis while simultaneously lending FCM high efficiency in the use of NURBS-based shape functions and a geometry description as suggested by the IGA. This paper is organized as follows: In the next Section 2 we give a short summary of the basic ideas of the FCM and IGA. Section 3 will connect FCM to Constructive Solid Geometry. Section 4 extends the approach to three-dimensional thin-walled structures using isogeometric analysis. In Section 5 we will demonstrate the high accuracy and efficiency of the method on a modification of a classical shell benchmark problem. We finally present a more complex example to reveal the relevance and potential for practical applications.

2. Basics of IGA and FCM

To provide a key to the notations employed, we will give a very short summary of the basic concept of IGA and the finite cell method for three-dimensional linear elasticity. For further details we refer to the cited literature on these topics.

2.1. Isogeometric analysis

The key idea of isogeometric analysis is to use the same basis functions for the representation of geometry in CAD and the approximation of solutions fields in the finite element analysis [47,1]. Due to their relative simplicity and ubiquity in today's CAD tools, isogeometric analysis is usually based on B-splines and non-uniform rational B-splines (NURBS).

A univariate B-spline basis of polynomial degree p consists of n basis functions $N_{i,p}(\xi)$, where $i = 1,\ldots,n$. It is generated from a knot vector Ξ, which is a non-decreasing sequence of coordinates in the parameter space ξ [4,48]

$$\Xi = \{\xi_1, \xi_2, \ldots, \xi_{n+p+1}\}, \quad \xi_1 \leqslant \xi_2 \leqslant \ldots \leqslant \xi_{n+p+1} \tag{1}$$

Piecewise polynomial B-spline basis functions are defined over $p + 1$ knot spans, which join smoothly up to a continuous differentiability