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Goal-adaptive Isogeometric Analysis with hierarchical splines

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ABSTRACT

In this work, a method of goal-adaptive Isogeometric Analysis is proposed. We combine goal-oriented error estimation and adaptivity with hierarchical B-splines for local *h*-refinement. The goal-oriented error estimator is computed with a *p*-refined discrete dual space, which is adaptively refined alongside the primal space. This discrete dual space is proven to be a strict superset of the primal space. Hierarchical refinements are introduced in marked regions that are formed as the union of chosen coarse-level spline supports from the primal basis. We present two ways of extracting localized refinement indicators suitable for the hierarchical refinement procedure: one based on a partitioning of the dual-weighted residual into contributions of basis function supports and one based on the combination of element indicators within a basis function support. The proposed goal-oriented adaptive strategy is exemplified for the Poisson problem and a free-surface flow problem. Numerical experiments on these problems show convergence of the adaptive method with optimal rates. Furthermore, the corresponding goal-oriented error estimators are shown to be accurate, with effectivity indices in the range of 0.7–1.1.

1. Introduction

Since the usage of computers has become widespread in engineering design, performing analysis on geometries designed using Computer Aided Design (CAD) software has been a common task. Isogeometric Analysis [1,2], is a framework for solving (partial) differential equations on domains generated by CAD software. It aims to eliminate or significantly reduce the time required for preparing the designed geometry for analysis, by directly using the CAD representation of the geometry in the analysis step. In addition to the benefits from an engineering management perspective, the exact representation of the design geometries in the analysis step has benefits for applications where the smoothness of the boundary of the domain plays a role, such as flow boundary layers and sliding contact of surfaces [1]. Possibility of having higher order differentiable solutions in Isogeometric Analysis has also proven to be very desirable for many engineering applications where continuity of the solution is a requirement due to the employed formulation, such as binary phase separation [3], gradient damage models [4] and analysis of shell structures [5,6]. Due to its positive attributes, the interest in Isogeometric Analysis has grown very rapidly after its initial introduction, with applications to many complicated engineering problems in material fracture [7,8], contact mechanics [9,10], turbulence computation [11,12], free-surface flows [13], fluid-structure interaction [14], shape optimization [15,16] and many others.

Although Isogeometric Analysis aims to overcome some of the problems often encountered in the engineering design-through-analysis process, the fact that the objects used for geometry representation are often not adequate for attaining a certain accuracy in analysis means that there has to be some sort of a preprocessing step. If this step is done heuristically,

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it again requires manual labor. To have a truly smooth work flow from design to analysis, adapting the initial design representations based on an automated procedure is essential. Adaptive Isogeometric Analysis methods based on error estimators are vital in performing this task with effectively no intervention by the engineer. Achieving this requires two issues to be addressed: Enriching the approximation space through local refinement of the basis, and finding reliable refinement indicators to assess if and where refinement should take place. The aim of this work is to address these issues by applying goal-oriented error estimation principles to develop an adaptive Isogeometric Analysis method using hierarchical B-splines as the local refinement technique.

In assessing CAD technologies in the context of adaptive Isogeometric Analysis, various qualities are important. Especially locality of the refinement procedure plays an important role, along with the requirement of linear independence and compatibility with the state of the art CAD technologies that are in use in the industry. Hierarchical splines [17] fulfill all of these requirements, yet they are constructed in a simple way. The hierarchical refinement method has been applied to B-splines [17–19] and NURBS [20]. In the field of engineering design, NURBS is the industry standard technology [2,21,22]. However, B-splines have many properties in common with NURBS and in terms of refinement techniques, virtually every concept can be extended from B-splines to NURBS with relative ease. Therefore, hierarchical B-splines are chosen to be used in this study.

Adaptive Isogeometric Analysis with hierarchical splines has already been studied using refinement indicators based on so called bubble functions [18] and gradients of the approximate solution [23]. In addition to these, hierarchically refined bases are used for Galerkin discretizations for several applications [24,25]. Adaptive Isogeometric Analysis has also been studied with T-splines [26,27], PHT-splines [28,29], LR-splines [30] and using common Finite Element Method (FEM)-type basis functions with spline parametrizations [31,32]. Adaptivity with a method of constructing locally refined B-spline elements ensuring up to C^1-conformity through a matching procedure had been investigated [33] even before the inception of Isogeometric Analysis.

Next to the techniques of constructing locally refined approximation spaces, an integral element of adaptive refinement is the refinement indicator that is used in deciding where and how a mesh needs to be refined. Refinement indicators are usually based on an a-posteriori estimate of the error. Among the most prominent a-posteriori error estimation methods for Galerkin discretizations are recovery-based methods, residual based methods and goal-oriented methods. For a review of various a-posteriori error estimation techniques, see [34,35]. The choice of goal-oriented error estimation is motivated by its very nature, i.e. estimating the error in a quantity that is of practical relevance for the application. In mesh adaptivity using estimators that do not take the quantity of interest into account, the refinement aims to resolve features of the solution with equal accuracy over the whole computational domain. In many problems the complicated features of the solution on some parts of the domain might not influence the quantity of interest. Hence, improving the approximate solution based on goal-oriented error estimators is a very natural choice, given that there is a clear quantity of interest.

This work considers the local h-refinement of Isogeometric Analysis spaces using hierarchical B-splines, with refinement indicators derived from goal-oriented error estimators. The main contributions of this work are a goal-oriented error estimator using adapted hierarchical B-spline primal spaces that are strictly nested within the proposed dual spaces, the construction of refinement indicators that are suitable for hierarchical refinement, the numerical investigation of these indicators and actual goal-adaptive Isogeometric discretizations of the two example problems.

The proposed methods are applied to the solution of exemplary elliptic problems of Poisson's equation and a prototypical free-surface flow problem. Both problems are posed on 2D domains, yet the methodology is formulated in general dimensions and in an operator-independent way wherever possible. Hence, extension to other elliptic problems should require minimal modifications to the proposed methodology. Furthermore, for problems in 3D, the benefits of local refinement in terms of computational cost would be even more relevant.

Starting with a brief introduction to B-splines, the employed refinement technique using hierarchical B-splines is reviewed in Section 2 in the light of adaptive Isogeometric Analysis. The method of goal-oriented error estimation is introduced in a general setting and an error estimator suitable for adaptive-refinement is proposed in Section 3. Furthermore, in Section 3 two refinement indicators are proposed using this estimator. In Section 4, the error estimator and refinement indicators are studied numerically and the adaptive procedure is tested for two problems. Conclusions and future recommendations are presented in Section 5.

2. Hierarchical refinement of B-splines

Given a bounded open interval $\hat{\Omega} \subset \mathbb{R}$, named the *parameter domain*, B-splines are piecewise polynomials $N_{i,p} : \hat{\Omega} \to \mathbb{R}$ of degree $p \in \mathbb{Z}_{\geqslant 0}$, that form a basis of a globally C^k-continuous polynomial space, with $-1 \leqslant k < p$. The space formed by the B-splines can be characterized by a non-decreasing sequence of *knots*, named the *knot vector*. Knots are points in $\hat{\Omega}$, at which the B-splines have reduced continuity, i.e. locations at which the basis functions are not C^∞-continuous. A *B-spline curve* $\mathbf{C} : \hat{\Omega} \to \Omega$ is a parametrization from the parameter domain $\hat{\Omega} \subset \mathbb{R}$ to the *physical domain* $\Omega \subset \mathbb{R}^{d'}$, with $d' \geqslant 1$, using the B-spline basis. The coefficients of the B-splines $\mathbf{B}_i \in \mathbb{R}^{d'}$ are referred to as the *control points*. The B-spline curve is described as:

$$\mathbf{C}(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi)\mathbf{B}_i,$$