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A strict error bound with separated contributions of the discretization and of the iterative solver in non-overlapping domain decomposition methods

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This paper deals with the estimation of the distance between the solution of a static linear mechanic problem and its approximation by the finite element method solved with a non-overlapping domain decomposition method (FETI or BDD). We propose a new strict upper bound of the error which separates the contribution of the iterative solver and the contribution of the discretization. Numerical assessments show that the bound is sharp and enables us to define an objective stopping criterion for the iterative solver.

1. Introduction

Developing robust numerical methods to solve systems of partial differential equations has become a major challenge in engineering. Indeed industrialists wish to adopt virtual testing in order to replace expensive experimental studies up to the certification of their structures. The massive use of virtual prototyping relies on the capacity to warrant the quality of the numerical solutions. In the context of the Finite Element Method (FEM), *a posteriori* error estimators permit to estimate the distance between the unknown exact solution and the numerical solution. Initial methods [1–3] evaluated globally the effect of the spatial discretization for linear problems, they have been extended to non-linear and time-dependent problems, and to the estimation of the error on quantities of interest. Another prerequisite for virtual testing is the ability to conduct large scale computations, because reliable models involve lots of degrees of freedom. Non-overlapping Domain Decomposition Methods (DDM) offer a favorable framework for fast iterative solvers adapted to modern clusters [4].

Most upper bounds for the error which do not involve constants rely on the computation of admissible stress and displacement fields. In a recent paper [5], the classical methods to construct statically admissible fields were extended to the framework of substructured problems. The estimator which ensues is fully parallel and totally integrated to classical DD solvers BDD [6] and FETI [7]. It provides a guaranteed upper bound whether the iterative solver of the interface problem has converged or not; unfortunately, it is not able to separate the different sources of error, namely the error due to the discretization and the error due to the lack of convergence at the interface. In [5], a Gamma-shape structure clamped on its basis and sollicitated in traction and shear on its upper-right side was considered. The structure was split into 8 subdomains

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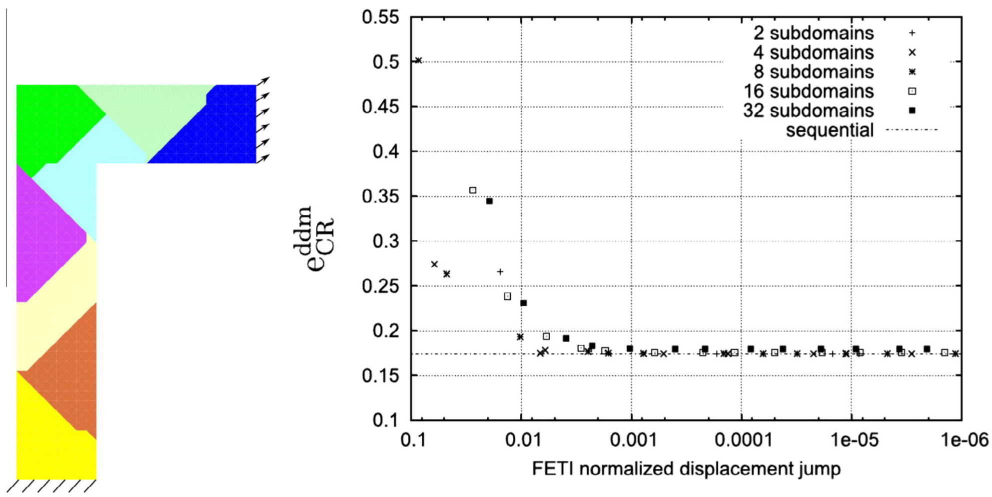
Fig. 1(a) and the problem was solved with classical DD solver. The error estimation provided by the estimator proposed in [5] is computed at each iteration of the DD solver (Fig. 1)(b). We clearly observe L-shaped curves highlighting the fast convergence rate of the estimator e_{CR}^{DDM} with respect to the domain decomposition residual. After a few iterations, the error due to the non-verification of the continuity and balance on the interface is insignificant compared to the contribution of the discretization error (which can be visualized by the estimator of the sequential problem). On that example, we observe that, whatever the substructuring, after 4 iterations there is no improvement of the approximation, while classical stopping criteria based on the decrease of a norm of the interface residual would imply 5 times more iterations.

Then in order to avoid oversolving, we wish to distinguish the contributions of the discretization and of the iterative solver to the estimation of the error. The non-convergence of the solver will be referred to as the algebraic error and no other sources of errors (rounding, representation of the loadings) will be considered.

Various articles have dealt with the separation of the contributions to the error and with the definition of new stopping criteria. In [8] the author insists on the use of the energy norm for the measurement of the residual instead of a classical Euclidean norm, in order to link the iterative methods to the properties of the approximated problem. In the framework of multigrid methods, an adaptive procedure to define both the refinement and the stopping criterion is developed in [9]. However, this technique demands the computation of constants since it is based on *a priori* estimates. The error in constitutive relation does not require the calculation of such constants and provides guaranteed upper bounds, it was applied to various problems that introduce other sources of error. In [10,11], the authors define a time error indicator to separate the part of the error due to the time discretization from the part due to the space discretization, and they use it to optimize the time steps. This work is extended in [12] in which an indicator of the effect of non-linear iterations complete the total error estimation. Finally, in the case of contact problems, the separation of the discretization error and of the algebraic error is performed for problems solved with the fixed-point method [13] and with a Neumann–Dirichlet algorithm [14]. Nevertheless, the computation of all those indicators requires the resolution of auxiliary problems which considerably increase the cost of the estimation (indeed, one has to compute statically admissible fields for each problem, which can be a costly step). For the finite volume method, the separation of the different sources of error and the definition of a new stopping criterion is exposed in [15] for second-order elliptic problems. This question of balancing the sources of error has also been addressed for error estimation on quantity of interest: in [16], a goal-oriented procedure to solve a problem with the multigrid method with a level of precision specified by the user is proposed; a similar approach is presented in [17] for the bound method [18,19].

In this paper, we present a new guaranteed upper bound that separates the algebraic error (represented by a well chosen norm of the residual) from the discretization error of the subdomains in the case of a linear problem solved with a classical DD solver (BDD or FETI). This separation enables us to define a new stopping criterion for the iterative solver. The method relies on the parallel procedures to build admissible fields proposed in [5] but compared to the estimator of that paper, the procedures are called much less often.

The paper is organized as follow. In Section 2 we define the reference problem, we recall the principle of the error in constitutive relation and we detail the error estimation in the substructured context by highlighting the fields created in the FETI and BDD solvers. In Section 3 we prove the new guaranteed upper bound separating the algebraic error and the discretization error on each subdomain and we explain how it leads us to define a new stopping criterion for the iterative solver. In



(a) Domain decomposition and loading

(b) Convergence of error estimator vs DD residual

Fig. 1. Error estimator from [5].