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A staggered approach to shape and topology optimization using the traction method and an evolutionary-type advancing front algorithm

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Abstract

We investigate a novel approach for structural shape optimization on the basis of complementary shape and topological sensitivity analysis. As in early approaches to shape optimization, the domain variation is specified by modification of boundary nodal points, hence leading to an updated Lagrangian description for the course of optimization. To overcome the formation of oscillating boundaries in the optimal design trials, we employ the traction method to establish smooth descent directions for shape variation. Therein, an auxiliary elastic problem is solved in which the shape sensitivity constitutes the external loading and the deformation of the auxiliary elastic body is used as a descent direction for shape variation rather than the shape sensitivity itself. We complement this method through an evolutionary-type element removal procedure that is based on the topological sensitivity such that an advancing front algorithm is gradually removing elements from the design boundary of the domain. Once the minimum topological sensitivity is no longer encountered at the design boundary, we create a hole in the domain, again using the topological sensitivity to specify its exact location, and resume the element removal procedure for the newly established design boundary. Since this approach yields only a vague estimate of the true optimal shape of newly established holes, the traction method is again used for shape variation of the extended design boundary.

Keywords: Shape optimization; Parameter-free; Topological sensitivity; Traction method

1. Introduction

In the field of structural optimization, the greatest domain variability is attributed to methods of topology optimization. As opposed to the sole variation of the domain boundary in classical shape optimization, methods of topology optimization do also take into account the possible creation of new holes within the domain. As a consequence, a greater design space is established in which it is possible to seek for optimal design trials that do not only meet the requirements of local but rather global optimality criteria since the limitation to a certain topological space of solutions is overcome.

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A well-established group of methods for topology optimization is characterized by the use of material homogenization or relaxation techniques. Approaches of this type date back to at least the early contribution by Bendsoe and Kikuchi [1] in which a material distribution problem is solved by means of composite materials that consist of two material constituents, namely phases filled by either substance or void. Basically, one seeks to determine the optimal layout of micro-structured cells whereby macroscopic constitutive equations, being obtained through homogenization, determine the structural response for the resulting distribution of microscopic material constituents. The layout of a micro-structured cell is obtained through placing a coherent quadratic shaped void phase at the cell centre whereby the edge-length of the void phase is determined through a local density function that represents the main design variable. The problem is solved on a fixed rectangular discretization such that no re-meshing during the optimization is to be considered even though this approach yields optimal design trials that exhibit a non-smooth boundary representation.

More recently, Allaire [2] gave a profound mathematical justification of the homogenization method and extended the concept of optimal microstructures to a three-dimensional setting. Numerically, the void phase is accounted for by use of a so-called *Ersatz material*[1] and the relaxed problem formulation is solved by an alternate direction algorithm. Composite designs that are present in the optimal design stage are removed by a penalization technique in a post-processing step if such a design is not desired.

Another type of material distribution approach to structural optimization has first been proposed in the contribution by Bendsoe [3] and is now commonly referred to as SIMP[2] method [4,5]. Therein, one does not attribute a certain microstructure to intermediate density values but rather follows a direct approach in which an artificial power law is employed in order to obtain a functional dependence between intermediate density values and the respective material stiffness coefficients. Through introduction of a penalization-like exponential parameter, the dependence function is forced to yield a sub-linear trade-off such that intermediate density values supply little stiffness at an unreasonable cost.

A conceptually different, so-called evolutionary structural optimization method, has been proposed by Xie and Steven [6] and further investigated in the contributions [7–9]. In this method, one aims at gradually removing ineffectively used material from the design domain which amounts to an element removal procedure that uses the strain energy density as a rejection criterion. Generally speaking, one considers an iterative procedure in which a certain number of elements that exhibit a relatively low strain energy density is removed from the domain and subsequently the design is re-evaluated. As a stopping criterion, either a desired volume fraction or a constraint on the structural response is to be specified. For the numerical implementation, the actual element removal approach is substituted by attributing an elastic modulus close to zero to all candidate elements, therefore allowing to capture the evolving structure on a fixed discretization. Several modifications and extensions of this approach have been investigated in the literature, such as considering rejection criteria on the basis of equivalent stress measures [10,11], the possibility of element (re-)activation in high-stress regions [10,12,13], and involving adaptive mesh refinement at the evolving boundary [14].

An increasing popularity is attributed to methods of topology optimization that rely on the powerful concept of level-set functions for domain variation [15,16]. In the contribution by Allaire [17,18] and the conceptually related work by Wang [19], the level-set method is used for front propagation where the velocity function is derived from classical shape sensitivity analysis. The level-set approach is characterized by a comparatively low computational cost since the set of admissible shapes is captured on a fixed Eulerian (hold-all) domain and using the *Ersatz material* approach to account for the void (or outer) phases of the evolving structure.

However, although providing a vast domain variability through the use of a level-set function, a critical aspect of this method lies in the fact that there exists no actual mechanism for creating new holes in the domain, at least considering a two-dimensional setting. Therefore, the optimal design trials depend on the choice of the initial configuration. This drawback can be overcome when not only considering the shape but also the topological sensitivity [20–24], a measure that represents the change in an objective functional on the condition that a small (circular) hole is created within the domain. Accordingly, this notion is not only applicable for the enrichment of optimization schemes that rely on a level-set function [25–29] but can also be employed in classical methods of structural optimization using a direct boundary representation through an underlying finite element discretization or on the basis of a parametrized geometry model.

An early representative of the latter type has been investigated by Eschenauer and is referred to as bubble method [30]. Therein, one considers a series of shape optimization runs on a hierarchical sequence of geometry models

[1] A material that is attributed with very low stiffness coefficients.

[2] 'Solid isotropic material with penalization' [4] or 'solid isotropic microstructures with penalization' [5].