Available online at www.sciencedirect.com
ScienceDirect
Comput. Methods Appl. Mech. Engrg. 287 (2015) 212–228
Computer methods in applied mechanics and engineering
www.elsevier.com/locate/cma

Strict bounding of quantities of interest in computations based on domain decomposition

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Received 31 July 2014; received in revised form 8 January 2015; accepted 13 January 2015
Available online 23 January 2015

Abstract

This paper deals with bounding the error on the estimation of quantities of interest obtained by finite element and domain decomposition methods. The proposed bounds are written in order to separate the two errors involved in the resolution of reference and adjoint problems: on the one hand the discretization error due to the finite element method and on the other hand the algebraic error due to the use of the iterative solver. Beside practical considerations on the parallel computation of the bounds, it is shown that the interface conformity can be slightly relaxed so that local enrichment or refinement are possible in the subdomains bearing singularities or quantities of interest which simplifies the improvement of the estimation. Academic assessments are given on 2D static linear mechanic problems.

Keywords: Verification; Non-overlapping domain decomposition methods; Quantity of interest; Local refinement

1. Introduction

In order to certify structures by computations, two fundamental abilities must be gathered: (i) conducting large computations which can be achieved by using non-overlapping domain decomposition methods to exploit parallel hardware [1–3], (ii) warrantying the quality of the results which implies to use verification techniques. The computed approximation is then completed by an estimation of its distance to the exact solution, that distance being either on a global norm [4–7] or on a selection of engineering quantities of interest [8–10].

This paper is the sequel of contributions by the authors in order to associate domain decomposition methods (DDM) and verification techniques. In [11] it was shown how admissible fields (aka balanced residuals) could be naturally computed in parallel when solving one finite element system with classical DDM solvers like FETI or BDD. This tool was necessary to obtain guaranteed error bounds. Moreover the proposed method encapsulated classical sequential

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http://dx.doi.org/10.1016/j.cma.2015.01.009
0045-7825/

techniques [12–15] that could be employed independently on the subdomains. In [16] a global upper bound of the error was separated in two: the first contribution was purely due to the iterative solver employed in DDM and it could be made as small as wanted by doing more iterations, whereas the second contribution depended on the discretization and it was quasi-constant during the iterations. This result enabled us to stop iterations when the overall quality of the approximation could not be improved, which was much earlier than classical criteria on the norm of the residual would have suggested.

The purpose of this paper is to extend these works to the handling of quantities of interest. First we show how estimation on quantities of interest can be conducted easily within DDM with a separation of the contribution of the solver and of the discretization. Second we demonstrate that all the wanted properties can be preserved with nested kinematics at the interface which simplifies the improvement of the estimation by local adaptation.

The question of separating the contribution of the solver and of the discretization, in goal-oriented error estimation, was addressed in several other studies with different approaches. In [17,18], the quantity of interest was the mean value of the unknown field in a region of the structure, the coarse mesh of the structure was seen as a decomposition of the true refined mesh which allowed to take advantage of the properties of FETI algorithm. In [19], the authors proposed constant-free asymptotic upper and lower bounds but they did not connect the iteration and discretization contributions. In [20], error indicators separating contributions were developed in the context of multigrids methods for Poisson and Stokes equations. They lead to a gain in terms of CPU time but the bounds were not guaranteed. Using the dual weighted residual method [10] for goal-oriented error estimation, a balance of discretization and iteration errors was proposed for eigenvalue problems in [21]. Finally, in [22], the authors proposed goal-oriented a posteriori error estimator for problems discretized with mixed finite elements and multiscale domain decomposition that separated the contributions of the discretization of the subdomains from the contribution of the mortar interface.

In this paper, we employ classical extractor techniques [23,24,9], which leads to the definition of an adjoint problem which is solved at the same time as the forward problem using a block Krylov algorithm [25]. Thanks to [11,16] we compute guaranteed upper and lower bounds of the quantity of interest at each iteration with the separation of the two sources of error for each problem (forward and adjoint), which enables us to drive the iterative solver by the error on the quantity of interest.

Moreover, since we consider quantities of interest defined on small supports, the adjoint problems have very localized solution. A simple strategy to improve the quality of the estimation is then to better the resolution of the adjoint problem either with a locally refined mesh [26] or using the partition of unity method and handbook techniques [27]. We show that within the framework of domain decomposition methods, it is possible to enrich or refine the kinematic of selected subdomains without impairing the exactness of the bound nor the ease of computation even if a certain incompatibility appears at the interface.

The paper is organized as follows. In Section 2, we present the goal-oriented error estimation within domain decomposition framework; we propose bounds on quantities of interest that separate the contributions of the discretization and of the solver. In Section 3, we show that nested interfaces can be employed which simplifies the improvement of the quality of the bounds by local refinement. Assessments are presented in Section 4 and Section 5 concludes the paper.

2. Error estimation on quantity of interest in substructured context

2.1. Substructured formulation of the forward and adjoint problems

2.1.1. Reference problems

Let $\mathbb{R}^d$ represent the physical space ($d = 2$ or 3). Let us consider the static equilibrium of a (polyhedral) structure which occupies the open domain $\Omega \subset \mathbb{R}^d$ and which is subjected to given body force $\underline{f}$ within Ω, to given traction force $\underline{g}$ on $\partial_g \Omega$ and to given displacement field $\underline{u}_d$ on the complementary part of the boundary $\partial_u \Omega$ ($\text{meas}(\partial_u \Omega) \neq 0$). We assume that the structure undergoes small perturbations and that the material is linear elastic, characterized by Hooke's elasticity tensor $\mathbb{H}$. Let $\underline{u}$ be the unknown displacement field, $\underline{\underline{\varepsilon}}\left(\underline{u}\right)$ the symmetric part of the gradient of $\underline{u}$, $\underline{\underline{\sigma}}$ the Cauchy stress tensor. In order to evaluate (localized) quantities of interest, we consider an adjoint problem loaded by a (linear) extractor $\tilde{l}$ with small support. Fields related to the adjoint problem will always be noted with a tilde (for instance adjoint displacement is $\underline{\tilde{u}}$).

Let $\omega \subset \Omega$ be an open subset of Ω. We introduce three affine subspaces, one linear space and one positive form: