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Variationally consistent eXtended FE model for 3D planar and curved imperfect interfaces

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ABSTRACT

We propose an eXtended Finite Element Method convergent to the asymptotic solution of a thin interface problem for both planar and curved imperfect interfaces in three dimensions. The main advantage over standard cohesive-zone models is the bulk-mesh size independence. With respect to standard eXtended Finite Element Method, in the proposed procedure, blending and quadrature sub-domains are not required. The focus is on the evaluation of the accuracy of the proposed approach in solving three-dimensional benchmark tests. The numerical results are compared with those available from analytical solutions and spring-like interface models.

1. Introduction

We propose a simple, variationally consistent and reliable three-dimensional finite element approach for modelling thin interfaces such as those occurring in particle reinforced composites where spherical and cylindrical coated inclusions are embedded within a matrix. Mechanical properties strongly depend on interfacial bonding quality. Hence the determination of the maximum stress at the particle surface as a function of the applied load and the adhesion parameters is of great interest.

Well established theoretical approaches such as the Eshelby approach for the "dilute" inclusion problem [1,2], and (generalized) self-consistent schemes for interacting particles are available [3,4]. Analytical solutions cannot however be obtained for any general geometry and material behavior. Variational principles involving the strain polarization tensor were formulated by Hashin in order to obtain bounds of the effective elastic moduli of the three-phase composite [5]. A simplification consists in replacing the interphase with an equivalent interface model. This simplifies the subsequent numerical implementation. For instance, Hashin [6] derived the connections between the properties of a curved isotropic interphase and a spring-like interface. Different types of interface conditions were derived through asymptotic methods by Klarbring [7], Bigoni et al. [8], Geymonat et al. [9], Benveniste and Miloh [10] among others. A different approach based on the convergence analysis of suitable variational formulations for vanishing thickness of a thin interphase was followed by Caillerie [11], Acerbi and Buttazzo [12], and Suquet [13]; a recent contribution was given by Lebon and Rizzoni [14]. In particular, Suquet [13] developed a "mechanically consistent" mathematical framework for the asymptotic analysis of thin layers.

In finite element formulations, interface finite elements can be either placed along the finite element boundaries [15–17] or embedded within the finite elements [17]. The latter alternative has the advantage that the geometry of the interfaces is

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independent of the mesh in contrast to standard interface finite element models. Based on the Partition of Unity Property of the finite element shape functions [18], the eXtended Finite Element Model (XFEM) is a broad spectrum technique for dealing with cohesive embedded interfaces. In the last years, the Authors have developed a variant, called Regularized XFEM approach. The regularized XFEM approach was applied to elastic and cohesive interfaces [19,20], and strain localization in two-dimensional problems [21,22]. Recent two-dimensional applications to the delamination problem of an FRP strip glued to a concrete block have shown an excellent comparison with experimental results [23]. The appeal of the regularized XFEM approach is that imperfect interfaces of any thickness can be directly modeled after a minimal modification of the traditional XFEM format. Moreover, the subdivision of the finite element into quadrature sub-domains which is required by the standard XFEM approach can be avoided. Three-dimensional implementation has been recently discussed in [24]. Despite the wide diffusion of PUFEM methodology and its XFEM variant for interface models [25–27], their connections with classic asymptotic analysis such as those by Suquet [13] seem not to have been fully investigated yet. We show here that a close link can be established.

In particular, imperfect interfaces are studied where discontinuous displacements fields across the interface and continuous traction vector occur. The theoretical framework is based on the variational formulation proposed by Suquet [13]. In Section 2, we present a generalization of the Suquet's variational formulation based on the following steps:

- The interphase problem is formulated, where the thick layer has finite and small thickness;
- We construct an approximated solution featuring the characters of the expected solution;
- The relevant zero-order problem is defined corresponding to an interface with zero thickness, continuous stress and discontinuous displacement;
- The constitutive law associated with the approximated solution is introduced converging to a spring-like interface law for vanishing layer thickness.

The finite element implementation naturally leads to a standard XFEM framework, as shown in Section 3. It is worth noting that three-dimensional problems are addressed here by means of the regularized XFEM approach for the first time. Comparisons with numerical and analytical spring-like solutions are shown in Sections 4.1, 5.1, 5.2 in order to check whether the proposed approach correctly reproduces the expected results. While the structural responses obtained through the usual interface two-dimensional finite elements are affected from a pathological dependency on the mesh size of the bulk elements [16,15,17], we show that the regularized XFEM approach is mesh size-independent. The effect of the coating on the stress concentration is shown to strongly depend on the ratio between the elastic modulus of the matrix and the coating.

The proposed formulation differs from previously proposed XFEM formulations such as those developed by Sukumar et al. [28], Belytschko et al. [29], Fries and Belytschko [30] for two main aspects. Firstly, here displacement jumps are regularized and a length scale is introduced in the equilibrium equations while preserving their local structure. A previously assessed advantage is that the structural results are independent of the mesh size of the strain localization band arising in elasto-damaging materials [21,22]. With respect to traditional XFEM with cohesive-crack laws [31–33], the proposed XFEM procedure is self-blended, namely the implementation of blending procedures is not necessary [34]. Moreover, quadrature procedures based on element sub-domains, whose computational burden in three-dimensions is especially high, are not required as all the stiffness terms contain continuous functions. Previously, we showed [23] that the identification of the material parameters of the coating is simpler in the proposed approach than the identification of the material parameters of a traction-separation law. Furthermore, the proposed variational formulation differs from XFEM formulations based on weak discontinuities such as those proposed by Abbas et al. [35] and Vandoren et al. [36], and those for composite laminates developed by Iarve et al. [37,38], who also obtained mesh-size-independent evaluations of the structural strength. The results obtained for the case of spring-like interfaces agree with those obtained by Zhu et al. [27], who developed an XFEM formulation for spring-like zero thickness interfaces.

Unlike in previous Authors' studies such as Benvenuti et al. [20], where one-and two-dimensional planar interfaces were considered, in the present work: (i) we present an original application of Suquet's asymptotic analysis to the construction of an XFEM-oriented, asymptotically consistent variational formulation, that is different from the variational formulation previously adopted in [20]; (ii) the link of the regularized XFEM with the well known spring-like Hashin's model is assessed for the first time and accurately motivated through comparisons with analytical and numerical solutions in the general case of three-dimensional curved interfaces. Moreover, in the derivation of the numerical results, adaptive quadrature is employed instead of Gaussian quadrature. This allows to accurately capture the steep gradients of the regularization function. Finally, numerical results are commented showing the necessity of introducing special variational formulations (like the one presented in this paper) when dealing with regularized Heaviside enrichments.

Notation: The symmetrized gradient of a vector field $\mathbf{a}$ is indicated by $\nabla_s \mathbf{a}$, while the symmetrized dyadic product of two vector fields $\mathbf{a}$ and $\mathbf{b}$ is indicated by $\mathbf{a} \odot \mathbf{b}$.

2. Variational formulation

We focus on the analysis of Suquet [13] of a thin interphase between two elastic bodies, as illustrated in Fig. 1. In a body Ω, a thin layer is assumed close to the plane $x_N = 0$. The middle surface of the layer, defined as