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A large deformation formulation for fluid flow in a progressively fracturing porous material

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ABSTRACT

A general numerical model has been developed for fluid flow in a progressively fracturing porous medium subject to large deformations. The fluid flow away from the crack is modelled in a standard manner using Darcy's relation. In the discontinuity a similar relation is assumed for the fluid flow, but with a different permeability to take into account the higher porosity within the crack due to progressive damage evolution. The crack is described in a discrete manner by exploiting the partition-of-unity property of finite element shape functions. The nucleation and the opening of micro-cracks are modelled by a traction-separation relation. A heuristic approach is adopted to model the orientation of the cracks at the interfaces in the deformed configuration. A two-field formulation is derived, with the solid and the fluid velocities as unknowns. The weak formulation is obtained, assuming a Total Lagrangian formulation. This naturally leads to a set of coupled equations for the continuous and for the discontinuous parts of the mixture. The resulting discrete equations are nonlinear due to the cohesive-crack model, the large-deformation kinematic relations, and the coupling terms between the fine scale and the coarse scale. The capabilities of the model are shown at the hand of some example problems.

1. Introduction

Since the work of Terzaghi [1] and Biot [2], fluid flow in a deforming porous medium has received considerable attention. Indeed, the subject is crucial for the understanding and the prediction of the physical behaviour of many systems of interest. Initially, research has focused on petroleum and geotechnical engineering [3]. More recently, the techniques have also been applied to biology and medical sciences. Studies have been carried out to understand the complexity of the structure as well as the physical processes in human soft tissues, e.g. blood perfusion [4], skin and subcutis [5] and cartilageous tissues including intervertebral discs [6].

Recently, a two-scale numerical model has been constructed for crack propagation in a deforming fluid-saturated porous medium subject to small strains [7–10]. The saturated porous material was modelled as a two-phase mixture, composed of a deforming solid skeleton and an interstitial fluid. Crack growth was initially modelled using linear-elastic fracture mechanics, but later also via a cohesive-zone approach, where the process zone is lumped into a single plane ahead of the crack tip. The opening of this plane is governed by a traction-separation relation. At the fine scale the flow in the crack was modelled as a viscous fluid using Stokes' equations for the open cracks in the linear-elastic fracture mechanics approach, while for the modelling with cohesive cracks the fluid flow in the crack was assumed to obey Darcy's law, but with a different, and evolving permeability due to the higher porosity inside the crack. Since the cross-sectional dimensions of the cavity formed by the crack are assumed to be small compared to its length, the flow equations can be averaged over the cross section of the cavity. The resulting equations provide the momentum and mass couplings to the standard equations for a porous material, which hold on the coarse scale. In order to allow for the nucleation and the propagation of cracks in arbitrary directions, irrespective of the structure of the underlying finite element mesh, the model exploits the partition-of-unity property of finite element shape functions [11], see also [12–15].

Soft tissues can experience large deformations. The small strain assumption then no longer holds. In this contribution, we therefore make the extension to a finite strain framework, introducing nonlinear kinematics in combination with a hyperelastic material response. Thus, we follow the motion of the solid skeleton using a Lagrangian description and express the momentum balance equations using this description. Subsequently, we write the mass balance equations identifying the spatial point as the instantaneous material point occupied by the solid phase. The resulting

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system of equations is nonlinear due to the cohesive-crack model, the geometrically nonlinear effect and presence of the coupling terms. A linearisation is applied to the system for use within a Newton–Raphson iterative procedure, and a weighted-time scheme is applied to discretise the system in the time domain.

The paper is ordered as follows. In the next section, the nonlinear kinematic relations for a fracturing porous material are elaborated. These relations are used to construct the linear momentum and mass balance relations in Section 3, complemented by constitutive relations for the mixture in the bulk as well as at the interface in Section 4. The spatial discretisation, which exploits the partition-of-unity property, is presented in Section 5, followed by implementation aspects in Section 6. The performance of the model is assessed in Section 7, followed by some concluding remarks.

2. Nonlinear kinematics

Fig. 1(a) shows a body crossed by a discontinuity $\Gamma_{d,0}$ in the reference or undeformed configuration. The body is divided by the discontinuity into two sub-domains, Ω_0^+ and Ω_0^- ($\Omega_0 = \Omega_0^+ \cup \Omega_0^-$). A vector $\mathbf{n}_{\Gamma_{d,0}}$ is defined normal to the discontinuity surface $\Gamma_{d,0}$ in the direction of Ω_0^+. The total displacement field of the solid skeleton $\mathbf{u}$ at any time t consists of a continuous regular displacement field $\bar{\mathbf{u}}$ and a continuous additional displacement field $\hat{\mathbf{u}}$:

$$\mathbf{u}(\mathbf{X},t) = \bar{\mathbf{u}}(\mathbf{X},t) + \mathcal{H}_{\Gamma_{d,0}}\hat{\mathbf{u}}(\mathbf{X},t), \tag{1}$$

where $\mathbf{X}$ is the position vector of a material point in the undeformed configuration and $\mathcal{H}_{\Gamma_{d,0}}$ is the Heaviside step function centered at the discontinuity and is defined as:

$$\mathcal{H}_{\Gamma_{d},0}(\mathbf{X}) = \begin{cases} 1, & \mathbf{X} \in \Omega_0^+, \\ 0, & \mathbf{X} \in \Omega_0^-. \end{cases} \tag{2}$$

From the displacement decomposition in (1), the deformation map $\Phi(\mathbf{X},t)$ for a body crossed by a discontinuity can be written as:

$$\Phi(\mathbf{X},t) := \mathbf{x}(\mathbf{X},t) = \mathbf{X} + \bar{\mathbf{u}}(\mathbf{X},t) + \mathcal{H}_{\Gamma_{d,0}}\hat{\mathbf{u}}(\mathbf{X},t), \tag{3}$$

where $\mathbf{x}$ is the position vector of a material point in the deformed configuration. The velocity of the solid constituent is defined as

$$\dot{\mathbf{x}} = \frac{\mathrm{D}\,\mathbf{x}}{\mathrm{D}t} = \mathbf{v}_s, \tag{4}$$

where the superimposed dot indicates the material time derivative which follows the motion of the solid. The deformation gradient $\mathbf{F}$ is obtained by taking the gradient of (3) with respect to the undeformed configuration:

$$\mathbf{F} = \bar{\mathbf{F}} + \mathcal{H}_{\Gamma_{d,0}}\hat{\mathbf{F}}, \quad \mathbf{X} \in \Omega_0 \setminus \Gamma_{d,0}, \tag{5}$$

with $\bar{\mathbf{F}} = \mathbf{I} + \nabla_0\bar{\mathbf{u}}$ and $\hat{\mathbf{F}} = \nabla_0\hat{\mathbf{u}}$.

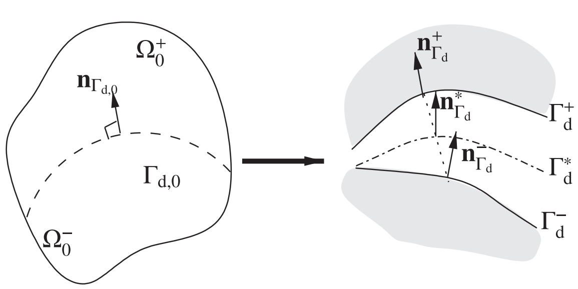

Fig. 1. (a) Schematic representation of body Ω_0 crossed by a material discontinuity $\Gamma_{d,0}$ in the undeformed configuration. (b) Discontinuity interfaces Γ_d^+ and Γ_d^- and their normal vector representation in the deformed configuration.

The volumetric change of the solid between the undeformed and deformed configuration is represented as $J = \det(\mathbf{F})$. Differentiation with respect to time yields

$$\dot{J} = J\,\nabla\mathbf{v}_s. \tag{6}$$

The magnitude of the displacement jump $\mathbf{u}_d$ at the discontinuity $\Gamma_{d,0}$ is represented as the magnitude of the additional displacement field $\hat{\mathbf{u}}$;

$$\mathbf{u}_d(\mathbf{X},t) = \hat{\mathbf{u}}(\mathbf{X},t), \quad \mathbf{X} \in \Gamma_{d,0}. \tag{7}$$

With aid of Nanson's relation for the normal $\mathbf{n}$ to a surface Γ

$$\mathbf{n} = J\mathbf{F}^{-T}\mathbf{n}_0\frac{\mathrm{d}\Gamma_0}{\mathrm{d}\Gamma}, \tag{8}$$

the expressions for the normals at the Ω_0^- side and at the Ω_0^+ side of the interface can be derived as

$$\mathbf{n}_{\Gamma_d}^- = \det(\bar{\mathbf{F}})\,\bar{\mathbf{F}}^{-T}\mathbf{n}_{d,0}\frac{\mathrm{d}\Gamma_{d,0}}{\mathrm{d}\Gamma_d^-} \tag{9}$$

$$\mathbf{n}_{\Gamma_d}^+ = \det(\bar{\mathbf{F}} + \hat{\mathbf{F}})\left(\bar{\mathbf{F}} + \hat{\mathbf{F}}\right)^{-T}\mathbf{n}_{d,0}\frac{\mathrm{d}\Gamma_{d,0}}{\mathrm{d}\Gamma_d^+}, \tag{10}$$

respectively. Fig. 1(b) illustrates the normal vector at the discontinuities. Considering the fact that the magnitude of the opening $\mathbf{u}_d$ will be relatively small, it is assumed that an average normal can be defined for use within a cohesive-zone model [14]:

$$\mathbf{n}_{\Gamma_d}^* = \det\left(\bar{\mathbf{F}} + \frac{1}{2}\hat{\mathbf{F}}\right)\left(\bar{\mathbf{F}} + \frac{1}{2}\hat{\mathbf{F}}\right)^{-1}\mathbf{n}_{d,0}\frac{\mathrm{d}\Gamma_{d,0}}{\mathrm{d}\Gamma_d^*}. \tag{11}$$

The vector $\mathbf{n}_{\Gamma_d}^*$ is used to define the traction vector at the 'average' discontinuity plane Γ_d^*, and to resolve a displacement jump into normal and tangential components. To simplify the notation, $\mathbf{n}_{\Gamma_d}$ will henceforth substitute $\mathbf{n}_{\Gamma_d}^*$.

3. Balance equations

We consider a mixture that consists of a solid skeleton with an interstitial fluid. There is no mass transfer between the constituents. The inertia effects, convective term and gravity acceleration are neglected and the process is isothermal. With these assumptions we write the balance of linear momentum for the solid and the fluid phases as:

$$\nabla \cdot \boldsymbol{\sigma}_\pi + \hat{\mathbf{p}}_\pi = \mathbf{0}, \tag{12}$$

where $\boldsymbol{\sigma}_\pi$ denotes the stress tensor of constituent π. In the remainder we will adopt $\pi = \mathrm{s,f}$, with s and f denoting the solid and fluid phases, respectively. Furthermore, $\hat{\mathbf{p}}_\pi$ is the source of momentum for constituent π from the other constituent, which takes into account the local drag interaction between solid and fluid. Considering that the latter source terms satisfy the momentum production constraint

$$\sum_{\pi=\mathrm{s,f}} \hat{\mathbf{p}}_\pi = \mathbf{0} \tag{13}$$

and adding the momentum balances for the solid and the fluid parts of the mixture, we obtain the balance of linear momentum for the mixture in the current, or deformed, configuration as:

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0}, \tag{14}$$

where the stress is composed of a solid and a fluid part:

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_s + \boldsymbol{\sigma}_f. \tag{15}$$

Now, defining the corresponding first Piola–Kirchhoff partial stress tensor as $\mathbf{P}_\pi = J\mathbf{F}^{-1} \cdot \boldsymbol{\sigma}_\pi$, the total first Piola–Kirchhoff stress tensor is given by