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Explicit finite deformation analysis of isogeometric membranes

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Abstract

NURBS-based isogeometric analysis was first extended to thin shell/membrane structures which allows for finite membrane stretching as well as large deflection and bending strain. The assumed non-linear kinematics employs the Kirchhoff–Love shell theory to describe the mechanical behaviour of thin to ultra-thin structures. The displacement fields are interpolated from the displacements of control points only, and no rotational degrees of freedom are used at control points. Due to the high order C^k ($k \geq 1$) continuity of NURBS shape functions the Kirchhoff–Love theory can be seamlessly implemented. An explicit time integration scheme is used to compute the transient response of membrane structures to time-domain excitations, and a dynamic relaxation method is employed to obtain steady-state solutions. The versatility and good performance of the present formulation are demonstrated with the aid of a number of test cases, including a square membrane strip under static pressure, the inflation of a spherical shell under internal pressure, the inflation of a square airbag and the inflation of a rubber balloon. The mechanical contribution of the bending stiffness is also evaluated.

Keywords: Membrane; Kirchhoff–Love shell; Isogeometric; NURBS; Explicit; Dynamic relaxation

1. Introduction

Thin membrane structures are ubiquitous in Nature from the lipid membrane of animal cells, virus shells, wings of flying insects, through the pleura (lung membrane), the vitelline membrane in eggs, the placenta of mammals, to the heart valves and the skin. A key feature of membrane structures is their high slenderness ratio and their ability to

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resist significant membrane stress whilst minimising their weight [1]. Due to their extreme efficiency in addressing particular types of environments and achieving outstanding functionalities, engineered membrane structures are also used for a wide range of applications such as building fabric structures (e.g. exhibition pavilions, permanent roofing structures), bioprosthetics (e.g. artificial heart valves), biomimetic (e.g. flying robots, artificial jelly-fish propulsion systems) and deployable structures (e.g. airbag, parachute, landing system of Mars Rover) [2,3].

The large slenderness associated with membrane structures means that the structural bending stiffness is generally very small and even negligible [1,4]. In the latter case, stress states characterised by negative principal components are not admissible. As a result, large deformations such as ballooning, local wrinkling or folding are produced to minimise the potential energy of the system [5].

The mechanical response of these very thin shell/membrane structures is more naturally described by the Kirchhoff–Love shell theory [6–8] incorporating membrane strains and change in curvature due to bending rather than full three-dimensional constitutive theories or Reissner–Mindlin shell theory [1] which include transverse shear stress. The accurate modelling of such thin structures under large deflection, rotation and strain is essential for understanding the mechanics of biological membranes (here, *membrane* refers to the biological meaning) [9–11] which is one of the motivations for the present study. The only practical way to analyse membrane structures featuring complex geometries is to use numerical methods such as the finite element method (FEM) [12–17]. The robust numerical simulation of membrane structures for arbitrary kinematics and non-linear constitutive laws is a challenging research area. Moreover, for practical applications, the design of membrane structures could be significantly streamlined by tying computer-aided design (CAD) and analysis together. Although the situation is rapidly evolving since the visionary work of Hughes et al. [18], engineering design and numerical analysis are traditionally not tightly integrated. The typical situation in engineering practice is that designs are encapsulated in computer-aided design (CAD) systems which are subsequently converted into finite element meshes through a, generally, time-consuming process which can be done at best semi-automatically. It is estimated that about 80% of overall analysis time is devoted to mesh generation in a number of industries such aerospace or automotive [19].

An important point for the computational analysis of Kirchhoff–Love shells is that C^1 continuity is required between elements [6,7,20,21] as curvature is a function of the second derivatives of the displacement, which is rather difficult to achieve for free-form geometries when using polynomial basis functions [6]. These polynomial shape functions, when applicable, introduce undesirably high order polynomials with inherent disadvantages such as oscillations in the discrete solution and costly numerical integration (see, e.g. Refs. [20,21], among many others). The computational burden associated with these approaches is particularly onerous in the presence of strong gradients in the solution and/or in the case of costly stress-update procedures at the quadrature-point level.

Rabczuk et al. developed mesh-free thin shell element formulations accounting for large deformations and fracture within partition of unity-enrichment [22,23]. Only the shell mid-surface was discretised (not the director field) and no rotational degrees of freedom were used. Rabczuk et al. [24] later extended their mesh-free approach to model fracture arising from fluid–structure interactions. Millán et al. [25] developed a robust automatic method based on smooth mesh-free maximum-entropy approximants to circumvent global parameterisation of surfaces of complex geometry and topology. The technique was applied to geometrically-exact thin shell problems. This method was recently extended by Millán et al. [26] to address shortcomings arising from overlapping of patches which resulted in redundant numerical quadrature, and so, computational cost.

Recently, the idea of using non-uniform rational B-Splines (NURBS) [27] as basis functions for analysis was introduced by Hughes et al. [18,19], and was named *isogeometric analysis*. In isogeometric analysis, the functions for the geometry description are used as basis functions for the analysis. Thus, the analysis works on a geometrically exact model. This offers the possibility to close the existing gap between design and analysis by merging design geometry and analysis model. It was demonstrated that not only were NURBS applicable to engineering analysis, but that they were better suited for many applications, and were able to deliver accuracy superior to standard finite elements (see, e.g., [28–36]). Subdivision surfaces and, more recently, T-Splines [37,38] and PHT-Splines [39–41], were also successfully employed in isogeometric analysis contexts. More importantly, NURBS are smooth, higher order functions which are used for geometric design and have become standard in CAD (computer aided design) programs. They allow great geometric flexibility and high order continuities at the same time. NURBS are therefore ideally suited as basis functions for Kirchhoff–Love shell [20,41–43].

Making use of these significant properties, Kiendl et al. [42,43] developed a NURBS-based isogeometric Kirchhoff–Love shell which can capture both geometrically linear and non-linear deformations. However, only