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Oscillatory Marangoni instability in a heated layer with insoluble surfactant adsorbed on the free surface

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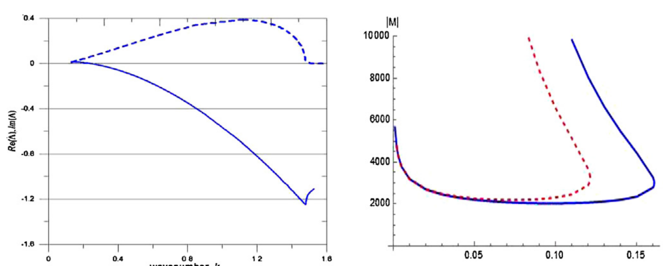
HIGHLIGHTS

- The presence of insoluble surfactant changes the type of instability by heating from below.
- By heating from above, the region of oscillatory instability is diminished by the surfactant.
- The high-frequency prediction for the critical number is valid only for very large *G*.

GRAPHICAL ABSTRACT

The onset of surface-tension driven convection in a heated liquid layer with insoluble surfactant adsorbed on the free surface is analyzed in the framework of the linear stability theory. The limit of a deep layer is considered. The general dispersion relation is obtained and investigated analytically and numerically.



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ABSTRACT

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1. Introduction

Onset of surface-tension driven convection in a viscous liquid under the action of thermal gradient is a paradigmatic example of oscillatory pattern formation in a nonequilibrium system [1]. Generally, the most typical mechanisms that produce an oscillatory instability of a motionless state are the mode mixing and the negative feedback with delay ("overstability").

A striking example of the oscillatory instability caused by mode mixing is the generation of waves by heating *from above*, discovered by Levchenko and Chernyakov [2] and thoroughly explored by Velarde and collaborators [3–5]. The origin of that instability is the linear mixing between two kinds of waves, (i) *transverse* (capillary-gravity) waves caused by a joint action of gravity and surface tension and (ii) *longitudinal* (dilational) waves driven by the gradient of surface-tension [6–8].

As an example of an oscillatory instability caused by negative feedback, one can mention instability in a viscous liquid film heated *from below* in presence of an insoluble surfactant [9–11]. Because of the advection of the surfactant by the liquid motion, the

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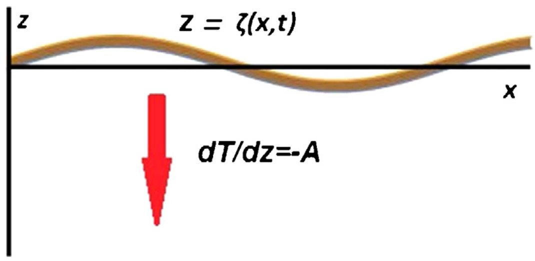

Fig. 1. Sketch of the problem geometry.

surfactant concentration decreases in the regions with divergent surface flow and increases in the regions with convergent flow. The tangential stresses driven by the surface tension inhomogeneity, that are directed opposite to the liquid motion, provide negative feedback that suppresses a monotonic instability but can lead to an oscillatory instability.

In the present paper, we investigate the onset of the Marangoni convection in the presence of an insoluble surfactant at the interface in the case of a "deep" layer, with the thickness large with respect to the capillary length. In Section 2, the problem is formulated, and the dispersion relation for waves is derived. In Section 3 we treat dilational waves generated by heating from below. In Section 4, the influence of the surfactant on the instability caused by mode mixing, is studied. Section 5 contains concluding remarks.

2. Formulation of the problem

We consider an incompressible Newtonian liquid that occupies a semi-infinite region $-\infty < x < \infty$, $-\infty < z < \zeta(x, t)$. Since we restrict ourselves to the linear stability of the system, it is sufficient to consider a two-dimensional problem. The free surface $z = \zeta(x, t)$ is covered by insoluble surfactant with concentration $\Gamma(x, t)$. The liquid is subjected to a vertical temperature gradient $-A$ (positive A for heating from below and negative A for heating from above), see Fig. 1. The surface tension σ of the liquid linearly depends on the temperature of the liquid T and on the surfactant concentration Γ,

$$\sigma = \sigma_0 - \sigma_1(T - T_0) - \sigma_2(\Gamma - \Gamma_0), \quad (1)$$

where $\sigma_1 = -\partial_T\sigma$, $\sigma_2 = -\partial_\Gamma\sigma$, whereas σ_0, T_0 and Γ_0 are, respectively, some reference values of surface tension, temperature, and surfactant concentration.

Conservation of momentum, mass and energy results in the following governing equations:

$$\mathbf{v}_t + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{\nabla p}{\rho} + \nu\nabla^2\mathbf{v} + \mathbf{g}, \quad (2)$$

$$\nabla\cdot\mathbf{v} = 0, \quad (3)$$

$$T_t + (\mathbf{v}\cdot\nabla)T = \chi\nabla^2 T; \quad (4)$$

$$-\infty < x < \infty, \quad -\infty < z < \zeta(x, t). \quad (5)$$

Here t is the time, $\mathbf{v} = (u, 0, w)$ is the velocity field, ρ is the density of the liquid, p is the difference between the local pressure and the atmospheric one, ν and χ are, respectively, kinematic viscosity and thermal diffusivity.

The distribution of the surfactant surface concentration Γ on the free surface $z = \zeta(x, t)$ is governed by the following equation:

$$\Gamma_t - \zeta_t(\mathbf{e}_z\cdot\nabla_s)\Gamma + \nabla_s\cdot(\mathbf{v}_\tau\Gamma) + (\nabla_s\cdot\mathbf{n})(\mathbf{v}\cdot\mathbf{n})\Gamma = D\nabla_s^2\Gamma, \quad (6)$$

where $\nabla_s = \nabla - \mathbf{n}(\mathbf{n}\cdot\nabla)$, $\mathbf{n} = (-\zeta_x, 1)(1+\zeta_x^2)^{-1/2}$ is the unit vector normal to the interface, $\mathbf{v}_\tau = \mathbf{v} - (\mathbf{v}\cdot\mathbf{n})\mathbf{n}$ is the tangential component of velocity, D is the surface diffusivity coefficient, and $\mathbf{e}_z = (0, 1)$ is a unit vector directed upwards. The first two terms in (6) characterize the temporal change of the surfactant concentration along the normal to the surface $z = \zeta(x, t)$ [12]. The third term describes advection of the surfactant by the surface flow. A change of the curvature radius results in variation of the surface area, which, in turn, alters the surface concentration. This effect is captured by the last term on the left side of (6) [13,14].

To close the system we require the balance of the normal and the tangential stresses, as well as the kinematic boundary condition:

$$z = \zeta(x, t): -p + 2\mu\mathbf{n}\cdot\mathcal{D}\cdot\mathbf{n} + 2H\sigma = 0, \quad (7)$$

$$2\mu\mathbf{n}\cdot\mathcal{D}\cdot\mathbf{t} = \nabla\sigma\cdot\mathbf{t}, \quad (8)$$

$$\zeta_t + u\zeta_x = w. \quad (9)$$

Here $\mu = \nu\rho$ is the viscosity, $\mathcal{D}$ is the deviatoric stress tensor, H is the mean interfacial curvature, σ is the surface tension, $\mathbf{t} = (1, \zeta_x)(1+\zeta_x^2)^{-1/2}$ is the unit tangential vector. We neglect the dilational and shear viscosities of the surface.

The basic state corresponding to the quiescent fluid is

$$u_b = w_b = 0, \quad \zeta_b = 0, \quad \Gamma_b = \Gamma_0, \quad p_b = -\rho g z, \quad T_b = -Az + T_0. \quad (10)$$

We study the stability of that state with respect to infinitesimal disturbances

$$u = u_b + \tilde{u}, \quad w = w_b + \tilde{w}, \quad \zeta = \zeta_b + \tilde{\zeta}, \quad \Gamma = \Gamma_b + \tilde{\gamma},$$

$$p = p_b + \tilde{p}, \quad T = T_b + \tilde{\theta}.$$

The governing equations for the disturbances are linearized around the base state (tildes are omitted):

$$u_t = -\frac{p_x}{\rho} + \nu(u_{xx} + u_{zz}), \quad (11)$$

$$w_t = -\frac{p_z}{\rho} + \nu(w_{xx} + w_{zz}), \quad (12)$$

$$u_x + w_z = 0, \quad (13)$$

$$\theta_t - Aw = \chi(\theta_{xx} + \theta_{zz}). \quad (14)$$

Linearizing the boundary condition on the free surface we obtain,

$$z = 0: \zeta_t = w, \quad (15)$$

$$-p + \rho g\zeta + 2\mu w_z = \sigma\zeta_{xx}, \quad (16)$$

$$\mu(u_z + w_x) = -\sigma_1(\theta_x - A\zeta_x) - \sigma_2\gamma_x, \quad (17)$$

$$\gamma_t + \Gamma_b u_x = D\gamma_{xx}, \quad (18)$$

$$\theta_z = 0. \quad (19)$$

Disturbances are located near the upper free surface and decay downwards, i.e.

$$u, w, \theta \to 0 \quad \text{at} \quad z \to -\infty. \quad (20)$$

We rewrite system (11)–(20) in the non-dimensional form using the following scales: the capillary length, $l_c = (\sigma/\rho g)^{1/2}$, is taken as a unit of length, l_c^2/χ as a time unit, χ/l_c as a velocity unit, $\rho\nu\chi/l_c^2$ as a unit of pressure, Al_c as a unit of temperature, and Γ_0 as a scale of the surfactant concentration. We arrive at the following system of equations and boundary conditions:

$$u_x + w_z = 0, \quad (21)$$

$$P^{-1}u_t = -p_x + u_{xx} + u_{zz}, \quad (22)$$

$$P^{-1}w_t = -p_z + w_{xx} + w_{zz}, \quad (23)$$

$$\theta_t - w = \theta_{xx} + \theta_{zz} \quad (24)$$

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