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A variational inequality formulation to incorporate the fluid lag in fluid-driven fracture propagation

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ABSTRACT

We have developed an efficient method to model the fluid lag in fluid-driven fracture propagation via a variational inequality formulation. The distinct feature of this method is that the configurations with and without a lag can be handled in a unified framework and no change of formulation is needed during the simulation at the time the fracturing liquid reaches the fracture tip. This is achieved by formulating the problem as solving for the non-negative pressure field in the fracture via a time-dependent (parabolic) variational inequality. Without introducing extra assumptions but merely based on mass conservation, this method is able to predict whether a fluid lag is going to remain or completely disappear as the fracturing progresses.

1. Introduction

The concerns of global energy shortage urge the development and improvement of energy technologies. Hydraulic fracturing is a technique applicable to both non-renewable and renewable energy technologies, as it can be used for both the extraction of oil and gas and the construction of enhanced geothermal systems. To understand the interplay between the local geological parameters such as the type of rock and its toughness and the distribution of natural fractures, the control parameters such as the flow rate and the choice of the fracturing fluid, and the resulting fracture path, researchers have spent efforts in modeling and developing computational methodologies for this fluid–solid interaction problem, see [1–8] and references therein.

The fracturing process may be controlled by either the toughness of the rock or the viscosity of the fracturing fluid, depending on the relative importance of these dissipation mechanisms. A detailed dimensional analysis of this competition is performed by Detournay [2], in which the author proposed a nondimensional constant which characterizes the distinct propagation regimes. An extension of this analysis is given by Garagash [3], in which the author suggested another non-dimensional constant which describes the progress of the fracture as early-time or late-time.

These non-dimensional constants also dictate the existence and evolution of a fluid lag, i.e., the dry zone between the fluid front and the fracture tip, as shown in Fig. 1. Incorporating the fluid lag in computational models without extra assumptions means to handle a contact-like boundary condition for the fluid front, and therefore poses challenges in numerical computations. Along this line, Hunsweck et al. [6] restricted themselves to the case of an always existing lag and devised a finite element method to explicitly track its evolution.

Nevertheless, with extra assumptions of the lag, researchers have obtained different behaviors of the pressure field. For example, Abé et al. [9] showed that a lag exists as long as the fracture is propagating. Based on this, many existing models

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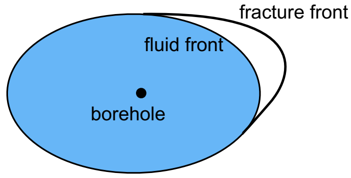

Fig. 1. Schematic illustrating the fluid lag and the different types of boundary conditions at different parts of the fracture front. The use of variational inequality eliminates the need of explicitly keeping track of the fluid front and the change of type of boundary condition along the fracture front.

and methods treat the lag as always infinitesimally small, e.g., [1,3,10]. Consequently, the predicted pressure profile usually has a nonphysical suction singularity near the tip. On the other hand, Bui and Parnes [11] pointed out that the singular pressure is a result of neglecting the transversal velocity near the tip, and taking into account this transversal velocity they obtained asymptotic finite pressure at the tip.

Here we propose a mathematical model for the fracturing process which also guarantees a finite, and hence physical, pressure solution up to the tip, but without taking into account the transversal velocity, and thus not needing to resolve the thickness dimension. Such finite pressure field is obtained by explicitly enforcing the pressure non-negativity as a constraint.

In this approach, the configuration with and without a lag are incorporated alike, and what dictates the lag's existence is mass conservation and pressure non-negativity. More precisely, we use a formulation based on a variational inequality to eliminate the need of switching the type of boundary conditions along different parts of the fracture front. Instead, we look for a nonnegative pressure field over the entire fracture surface, the lag being the region with a zero pressure. Therefore, this formulation permits studying the different regimes of fracture propagation regarding the existence of the lag. We observe that in the case of viscosity-controlled fracture propagation, a lag tends to form and persist; on the contrary, when the rock is very tough, the lag tends to disappear.

We are aware of the different conventions of the zero value of the pressure among researchers working on similar topics, some allowing negative pressure values. Here we use the symbol p to denote the fluid pressure, and thus always non-negative. The net pressure under compressive far-field stress σ_0 is thus given by $p - \sigma_0$. We use p as the primary variable to ease the formulation.

The organization of the subsequent sections is as follows. Section 2 introduces the governing equations for a typical fracture problem. Section 3 formulates the variational inequality and its numerical approximation for a straight fracture with symmetry in an infinitely tough solid. Section 4 then generalizes the formulation to a generic curvilinear fracture with a finite toughness. After that, in Section 5 we perform a numerical study in different propagation regimes of fracture. Finally, Section 6 offers a discussion of the numerical results.

2. Problem statement

For convenience we will state our problem in two-dimensions (plane-strain) according to the Khristinaovic–Geertsma–de Klerk (KGD) model [12,13], and the generalization to three-dimensions is straightforward. Furthermore, since it is not our intention to discuss curvilinear fracture propagation, we will assume that the fracture propagates along a known path.

We consider an infinite impermeable isotropic solid as rock mass without body forces that initially occupies $\mathbb{R}^2$ except $\Gamma = \Gamma(0) \subset \mathbb{R}^2$, a rectifiable and simple curve that represents the fracture in the undeformed configuration of the solid. Part or all of the fracture Γ may be pressurized by a liquid, and the liquid is kept being pumped to the system. As a result, the fracture is propped open by the liquid and will propagate before or after the liquid has covered all parts of the fracture. We wish to compute the evolution of $\Gamma = \Gamma(t)$ by solving the fluid–solid interaction problem. Below we summarize the governing equations.

Governing equations of the solid. We let $\Omega := \mathbb{R}^2 \setminus \Gamma$ denote the computational domain for the solid. With some origin chosen near or on Γ, we setup a Cartesian coordinate system (x, y), and define $R := \sqrt{x^2 + y^2}$.

In the absence of body forces, the governing equations for the solid read:

$$\operatorname{div} \boldsymbol{\sigma} = 0, \quad \text{in } \Omega, \tag{1a}$$

$$\boldsymbol{\sigma} = \lambda(\operatorname{div} \mathbf{u})\, \mathbb{I} + 2G\nabla^{\text{sym}}\mathbf{u}, \quad \text{in } \Omega, \tag{1b}$$

$$\boldsymbol{\sigma} \to -\sigma_0 \mathbb{I}, \quad \text{as } R \to \infty, \tag{1c}$$

$$\boldsymbol{\sigma} \cdot \mathbf{n}^{\pm} = -p\mathbf{n}^{\pm}, \quad \text{on } \Gamma^{\pm}, \tag{1d}$$

where $\boldsymbol{\sigma} : \Omega \to \mathbb{R}^{2\times 2}$ is the stress field, $\mathbf{u} : \Omega \to \mathbb{R}^2$ is the displacement field, ∇^{sym} denotes the symmetric part of the gradient, λ and G are Lamé constants of the solid that satisfy $\lambda + 2G > 0$ and $G > 0$, $\mathbb{I}$ is the 2×2 identity tensor, and $\sigma_0 \geqslant 0$ is the magnitude of the remote confining stress.