Comput. Methods Appl. Mech. Engrg. 272 (2014) 160–180

Contents lists available at ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Fifteen node tetrahedral elements for explicit methods in nonlinear solid dynamics

Kent T. Danielson *

Research Group, Engineering Systems and Materials Division, Geotechnical and Structures Laboratory, U.S. Army Engineer Research and Development Center, 3909 Halls Ferry Road, Attn: CEERD-GM-R, Vicksburg, MS 39180-6199, USA

ARTICLE INFO

Article history:
Received 11 September 2013
Received in revised form 8 January 2014
Accepted 12 January 2014
Available online 21 January 2014

Keywords:
Nonlinear solid dynamics
Second-order finite elements
Tetrahedron
Explicit time integration
Mass matrix diagonalization
Mass lumping

ABSTRACT

Despite the ease in meshing and benefits for modeling flexure, curved shapes, etc., second-order tetrahedral elements are not contained in typical explicit solid dynamic programs. This is primarily due to the lack of both a satisfactory consistent nodal loading distribution and mass lumping technique. Row summation lumping, for instance, produces negative vertex node masses for the popular ten node "serendipity" tetrahedron, which also has zero vertex node loads resulting from a constant pressure on an element face. This has led to piecewise composites of four node tetrahedrons to represent a ten node one in explicit codes. In this paper, truly second-order fifteen node formulations for compressible and for nearly incompressible materials are presented and evaluated. In addition to producing all positive nodal loads from a uniform traction, row summation mass lumping for the fifteen node element is shown to produce all positive nodal masses. Performance is assessed in standard benchmark problems and practical applications using various elastic and elastic–plastic material models and involving very large strains/deformations, severe distortions, and contact-impact. Comparisons are also made with several first-order elements and second-order hexahedral formulations. The offered elements performed satisfactorily in all examples. As recently found for second-order hexahedral elements, it is shown that the inclusion of face and centroidal nodes is vital for robust performance with row summation lumping, and high-order quadrature rules are crucial with explicit methods. These second-order elements are shown to be viable for practical applications, especially using today's parallel computers. Whereas the reliable performance is generally attained at significant computational expense compared with first-order and brick types, these elements can be more computationally competitive in flexure and have the desirable trait that they are amenable to automatic tetrahedral meshing software.

1. Introduction and background

Tetrahedral (Tet) finite elements are attractive because of their ease with automatic mesh generators, and the explicit method is an increasingly popular technique that is significantly used by analysts in many industries such as defense, crashworthiness, and metal forming. The explicit lumped mass approach without the use of a stiffness matrix is well suited for rapidly changing, high-rate, short duration applications, but can produce distinct nuances and severely affect element performances differently than in typical static/implicit methods. As a result, popular production software for explicit analysis has historically been limited to first-order elements. Various methods for enhanced analysis of important classes of problems

* Tel.: +1 (601)634 2039 (office); fax: +1 (601)634 2642.
E-mail address: Kent.T.Danielson@us.army.mil

0045-7825/$ - see front matter Published by Elsevier B.V.
http://dx.doi.org/10.1016/j.cma.2014.01.012

have emerged, such as Arbitrary Lagrangian Eulerian (ALE) (e.g., [1]), meshfree/particle (e.g., [2]), enrichment (e.g., [3]), discontinuous Galerkin (e.g., [4]), multiscale approaches, (e.g., [5]), etc. Classical finite element analysis, however, still remains as the primary computational method of choice for most solid mechanics applications, and these enhanced methods are frequently used in combination with finite elements, such as conversion to meshfree interpolants or enrichment in regions of interest. Therefore, improved finite element capabilities continue to be important.

Common meshing software can automatically generate or adaptively rezone unstructured finite element models with well-shaped tetrahedrons. Unstructured tetrahedral meshes typically are computationally expensive, as they consist of many more elements than in a hexahedral mesh (frequently by considerably more than an order of magnitude). The reduced meshing time for the analyst, however, can be well worth this computer cost, and the increased number of elements does permit more discontinuous solutions, e.g., more points to track a damage front or tearing due to an impacting penetrator. Unfortunately, standard first-order displacement-based (irreducible) tetrahedral elements generally perform very poorly in unstructured meshes, such as pathologically exhibiting shear locking in flexure or volumetric locking with nearly incompressible material models, e.g., metal plasticity or rubber hyperelasticity. Volumetric locking occurs when the number of incompressibility constraints is too large for the given number of nodal degrees of freedom [6]. At the element level, the order of interpolation for the pressure/volumetric strain is too high that causes it to be overly stiff, which can lead to extremely large errors and render the analysis useless. Being a four node constant pressure element, a reduction cannot be applied to either the order of integration or interpolation of the volumetric terms, so as to permit a first-order tetrahedron to stand on its own without locking in nearly incompressible situations. To remedy this problem, several special arrangements in hexahedral geometries, one with six non-symmetric Tets to average volumetric terms and another with twenty-four symmetric Tets without averaging, have been shown to avoid severe volumetric locking [7]. This permits the use of unstructured hexahedral volumes in the mesh, but now precludes the use of highly desirable automatic Tet meshers. Powerful Hex meshers exist and continue to improve, but they are still not as simple and robustly automatic as typical Tet meshing software. A more common alternative is Hex-Dominant meshing approaches that produce meshes primarily of bricks, but with noncritical fill regions of wedge, tetrahedra, and/or pyramid elements. As a result, others have used more general pressure averaging approaches, e.g., [8,9], or have developed mixed fluid mechanics type tetrahedrons, e.g., [10], whereby pressure is a nodal variable and is thus continuous across element boundaries. The nodal pressure degree of freedom essentially performs a pressure averaging over a group of tetrahedral elements to avoid acute locking in unstructured meshes, but may also complicate multi-material models or contact algorithms that must now accommodate the pressure at interfaces. Finally, the volumetric locking situation may be improved, but none of these first-order tetrahedral approaches address the shear locking that may occur in flexure.

In contrast, second-order ten node tetrahedral formulations for static/implicit formulations, e.g., [11–13], perform well in unstructured meshes both for nearly incompressible materials and in moderately thin structural flexure, and have been long contained in common production software, e.g., TEXGAP-3D [14] and ABAQUS [15]. These "serendipity" elements contain four vertex nodes and six midedge nodes to provide a geometrically complete second-order nodal interpolation and a trilinear variation of strain. Second-order elements can inherently represent curved shapes and naturally model bending accurately, particularly for thick to moderately thin beams, plates, and shells, but without using artificial hourglass control or adding any special modes or treatments. Even higher-order elements, e.g., cubic, quartic, etc., can further reduce shear locking, but there becomes a point of diminishing returns on interpolation order, particularly for explicit methods where the time increment size is related to nodal spacing. While performing well as flexural elements, second-order ones still maintain their versatility as solid elements. In contrast to structural elements, they can innately model multiple surface contact, such as self contact or front-back contact for the same element. With linear or higher stress/strain representations, selective reduced integration or reduced order (mixed/hybrid) pressure formulations (without nodal pressure variables) can now be used to effectively address volumetric locking on a single element basis for use in general unstructured meshes [14]. Unfortunately, ten node second-order tetrahedrons produce negative lumped masses, via row-summation or nodal integration, which effectively eliminates their usefulness in explicit methods. In addition, a traction applied to a six node elemental face produces zero nodal forces at the vertices with all of the loading thus being applied at the midedge nodes—a dilemma when the force is used to invoke a tensile release rule for a contacting node. Furthermore, the vertex nodes in a static or implicit analysis are coupled to the midedge ones in the stiffness matrix and are thus affected by the midedge nodal loads in each increment. Since the nodes are entirely uncoupled in each increment of an explicit solution method by the mass lumping, the influence of the midedge nodal loads is felt only at the vertex nodes in the internal force computation of the next time increment.

Several popular explicit solid dynamics codes, Presto [16], EPIC [17], Abaqus [15], IMPETUS AFEA [18], and LS-DYNA [19], contain ten node tetrahedrons. This can be accomplished by a modified formulation [16,20], whereby the effects of certain face or midedge nodes are applied in a weighted least squares manner to provide a uniform strain enhancement to first-order elements, which also requires hourglass control. Another approach is to use a composite element [21,22], whereby a ten node tetrahedron is converted into a group of first-order elements with strains and/or pressure averaged over the group. In Ref. [7], a group of eight first-order sub-elements is used with a weighted fraction of the original individual irreducible element value and the composite average used for each sub-element. By still using first-order elements, the well-defined lumped masses and contact tractions are naturally produced for the group. Incompressibility locking is treated by the volumetric averaging without using nodal pressure variables, and automatic Tet meshers are applicable. These are not truly second-order tetrahedrons, however, as the interpolation is piecewise first-order—the edges cannot be curved. In addition, its