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Non-ordinary state-based peridynamic analysis of stationary crack problems

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ABSTRACT

An implicit implementation of the non-ordinary state-based peridynamics formulation for quasi-static linearly elastic solids is presented. Emphasis is placed on assessing the accuracy of the numerical scheme in the vicinity of the crack front and other sources of stress concentration. We also present a comparative study of methods used to control the zero-energy modes inherent in the nonlocal definition of the strain tensor and reduce the spurious oscillations present particularly in regions of high strain gradients. The accuracy of the peridynamics scheme, including the impact of the lattice spacing and configuration, is assessed by performing an analysis of the near-tip stress and displacement fields (K-fields) for 2D problems. The manuscript also summarizes a verification study based on the classical 3D penny-shaped crack problem and a validation study of a 3D notched fracture specimen.

1. Introduction

An active and persistent challenge in computational mechanics is the modeling of fracture problems for both static cracks and arbitrary crack growth. In continuum mechanics, boundary value problems are based on partial derivatives with respect to the spatial coordinates. However, the assumption of continuity is inherently insufficient for modeling cracks as partial derivatives are undefined along the crack faces where the displacement field is discontinuous. Consequently, computational methods involving displacement gradients or higher-order spatial derivatives in a domain containing a crack must remove the discontinuous displacement field by either redefining the discretized body so that the crack lies on the boundary, or by using other techniques for evaluating the spatial derivatives on crack surfaces [1–3]. In contrast, the peridynamics formulation eliminates the spatial derivatives altogether by solely depending on an integral formulation of the force acting at a continuum point, resulting in equilibrium equations that are valid everywhere in the body.

The peridynamics (PD) theory was first introduced by Silling [4] as a continuum model for handling the spontaneous formation, propagation, branching and coalescing of discontinuities such as cracks. In PD, particles are influenced by other particles separated over a finite distance referred to as the *horizon*; hence the method is classified as a nonlocal theory. The use of nonlocal elasticity for the study of fracture problems is not new; an early study by Eringen et al. [5] presented the nonlocal solution for the Griffith crack problem and showed that the use of nonlocal elasticity captured the physical nature caused by the geometrical discontinuity of the crack, namely the removal of the crack-tip stress singularity, and led to a new stress-based fracture criteria incorporating the effects of the intermolecular forces as the dominant phenomena around

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the discontinuities. However, many of the past nonlocal theories rely on averaging the strains or stresses within the neighborhood of a point and then differentiating the stress tensor in the equations of motion. A detailed overview on the development of nonlocal theories and their application is given by Bažant and Jirásek [6].

The first PD formulation introduced in the literature is the so-called "bond-based" formulation, in which the continuum is discretized by a lattice of particles interacting through pair-wise bonds. In this formulation, the response of a bond is independent of the other bonds. As a consequence, the bond-based PD scheme is limited to constitutive models with a Poisson's ratio of 1/4. Nevertheless, the bond-based method has been and continues to be employed to study a wide variety of mechanical systems. Fracture and damage simulations have included the impact damage of composites [7,8], fracture of plain and reinforced concrete structures [9,10], fracture of membranes and fibers [11], and fracture of quenched glass [12]. At the nano-scale, PD has been used to model nanofiber networks and carbon nanotube-reinforced composites [13] as well as modeling nano-indentation of ultra-thin films in which the PD results compare favorably with those obtained using molecular dynamics and AFM experiments [14].

A reformulation of the bond-based PD relations proposed by Silling and coworkers [15,16] led to two *new* PD formulations referred to as *ordinary state-based* (OSB) and *non-ordinary state-based* (NOSB) PD. The state-based formulation introduces a force-vector state $\underline{\mathbf{T}}$ which maps a deformation-state into a force-state at all points within a volume of influence with no restrictions on the mapping function being linear or continuous [17]. Consequently, the state-based formulation eliminates the bond-based PD restriction requiring a Poisson's ratio of 1/4 for isotropic linear elastic materials since the bond forces now depend on the *collective* deformation of the bonds in the volume of influence. The OSB formulation is an active area of research and has been used to model problems involving plastic [18] and viscoelastic [19] constitutive models.

In contrast to its OSB counterpart, the NOSB PD formulation represents $\underline{\mathbf{T}}$ in terms of strain and stress tensors and consequently does not necessarily require co-linearity between the continuum points. The NOSB formulation therefore allows for classical continuum mechanics quantities, such as deformation gradient and stress tensor, to be used in constitutive models. This in turn allows for the incorporation of classical constitutive models into PD without the need to reformulate the constitutive laws in terms of the force-vector state $\underline{\mathbf{T}}$. Efforts to date using the NOSB formulation are relatively few and have focused mainly on the explicit formulation for dynamic simulations. In particular, Foster et al. [17] modeled rate-dependent plasticity for explicit dynamic Taylor impact tests of aluminum. Warren et al. [20] simulated transient dynamic fracture of a center cracked aluminum bar. Littlewood used the non-ordinary formulation to model fatigue crack growth of an elastic inclusion in a single elastic-viscoplastic crystal [21], and the dynamic fracture of an expanding steel tube was modeled using a classical mechanics elastic–plastic constitutive law [22]. Lastly, Tupek et al. [23] implemented a classical continuum damage model within the state-based formulation by modifying PD's influence function according to the accumulated damage state, where the bonds are severed within a horizon in accordance with the damage law.

In this manuscript, we develop a small-strain linearly elastic static implementation of the NOSB PD formulation and assess the ability of the numerical method to capture key *local* fracture parameters commonly affiliated with non-propagating crack problems, including the stress intensity factor and the stress and strain concentrations around notches. A review of the NOSB continuum formulation is presented in Section 2.1, while the new quasi-static implicit discretized formulation and numerical implementation for linear elastic small strain are presented in Section 2.2. In Section 3, schemes for controlling zero-energy modes inherent in the nonlocal definition of the strains are discussed. The effects of key formulation and implementation parameters such as the horizon, influence function and lattice structure, on the accuracy of the extracted near-tip fracture parameters are discussed in Section 4. Finally, Section 5 is dedicated to a verification study involving the classical 3D fracture problem of a penny-shaped crack, and to a validation analysis based on experimental results obtained on a notched specimen.

2. Quasi-static non-ordinary peridynamics formulation and algorithms

In preparation for the derivation of the implicit non-ordinary state-based PD scheme, a brief review of the critical notations and the underlying PD theory is needed. Section 2.1 is a synopsis of the PD conventions introduced by Silling et al. [4] and a review of the continuum non-ordinary formation as presented by Warren et al. [20]. In Section 2.2, the discretized continuum equations presented in Section 2.1 are expanded for the implicit quasi-static formulation, and the numerical implementation is presented.

2.1. Continuum formulation

A continuum point at $\mathbf{x}$ in domain $\mathcal{B}$ interacts with its neighbors, i.e. those material points located within a distance called the *horizon*, $\mathcal{H}$, by means of bonds between continuum points (Fig. 1). The reference position vector state $\underline{\mathbf{X}}$ is defined as

$$\underline{\mathbf{X}}\langle\xi\rangle = \xi = \mathbf{x}' - \mathbf{x}. \tag{1}$$

By operating on the bond ξ between material points at $\mathbf{x}'$ and $\mathbf{x}$, the deformation vector state $\underline{\mathbf{Y}}$ is the deformed state of the bond defined by

$$\underline{\mathbf{Y}}\langle\mathbf{x}' - \mathbf{x}\rangle = \boldsymbol{\eta} + \xi = (\mathbf{u}' + \mathbf{x}') - (\mathbf{u} - \mathbf{x}) \tag{2}$$