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A level set method for shape and topology optimization of both structure and support of continuum structures

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ABSTRACT

We present a level set method for the shape and topology optimization of both structure and support. Two level set functions are used to implicitly represent a structure. The traction free boundary and the Dirichlet boundary are represented separately and are allowed to be continuously changed during the optimization. The optimization problem of minimum compliance is considered. The shape derivatives of both boundaries are derived by using a Lagrangian function and the adjoint method. The finite element analysis is done by modifying a fixed background mesh, and we do not use the artificial weak material. Numerical examples in two dimensions are investigated.

1. Introduction

A complete definition of a structure comprises four ingredients: material, geometry, support, and load. Each of the ingredients affects a structure's performance. Therefore, optimization of any one of them will be helpful to improve the performance of a structure. In fact, structural optimization that focuses on the optimal design of these ingredients has been extensively studied for the past decades. In most cases, the design variable in structural optimization is the geometry of a structure or the distribution of material in a given space domain. In more advanced optimizations, two or more ingredients of a structure are simultaneously optimized, for instance the material and the geometry, the geometry and the support. Readers are referred to [1] and the references therein for a review.

The effects of the support on a structure's performance have been noticed and considered in optimization for a long time. In the early pioneering research work [2–9], the optimal position and stiffness of discrete supports of one dimensional structures, for instance a beam or a column, were studied, and in some of these researches the geometry of a structure was also optimized. The design objectives of these studies included yield moment, minimum compliance, Euler buckling load, costs of material, and etc. Usually, the costs of supports were assumed as position-dependent, and the criteria for the optimal locations of supports were derived. These studies were thereafter extended to the optimal design of discrete supports of two dimensional frame and truss structures [10–15].

Also, the optimal design of support was investigated for continuum structures. In these studies, the methods of optimization are divided into two categories. The first category is based on a field of background springs that represent potential supports [16–18]. A continuous design variable is given to each spring. When the value of a spring variable reaches the lower bound, the point where the spring is attached to the structure is considered as free. On the other hand, when the value of a

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spring variable reaches the upper bound, the attachment point is considered as fixed. Penalization of intermediate value of spring variables is often applied. The second category is based on continuous variation of support point or support boundary [19–22] where the supports are represented via geometric methods.

In the present study, a level set method [23–27] is developed for the shape and topology optimization of continuum structures and the supports, i.e., traction free Neumann boundary and Dirichlet boundary. Both the traction free boundary and the Dirichlet boundary are allowed to be continuously changed during the optimization. The optimization problem of minimum compliance is considered in the present study because its mathematical properties are well-known. The shape derivatives of both boundaries are derived by using a Lagrangian function and the adjoint method.

Although the focus of the present paper is the shape and topology optimization of continuum structures and their supports, the proposed level set method will by no means be restricted to this specific problem. In fact, it is a simple and versatile method to represent and propagate different segments of boundary of a geometric shape. It will be extended to deal with other problems where different boundary segments of a shape should be treated differently, for instance the optimization of a structure under design-dependent pressure loading. It may also find applications in image processing, geometric and physical modeling of solids and fluids.

2. Optimization problem

A shape $\Omega \subset \mathbb{R}^d$ ($d = 2$ or 3) is an open bounded set occupied by material. It is required that during optimization all admissible shapes Ω stay in a fixed reference domain $\mathcal{D} \subset \mathbb{R}^d$, i.e., $\Omega \subset \mathcal{D}$. The boundary of Ω consists of three disjoint parts

$$\partial\Omega = \Gamma_N \cup \Gamma_H \cup \Gamma_D$$

where a Neumann boundary condition is imposed on Γ_N, a homogeneous Neumann boundary condition, i.e., traction free, on Γ_H, and a Dirichlet boundary condition on Γ_D. In the present study, both Γ_H and Γ_D are subjected to optimization, and Γ_N is fixed.

The displacement field u in Ω is the unique solution to the equation

$$\begin{cases} -\operatorname{div}\sigma(u) = f & \text{in } \Omega \\ \sigma(u)\,n = t & \text{on } \Gamma_N \\ \sigma(u)\,n = 0 & \text{on } \Gamma_H \\ u = 0 & \text{on } \Gamma_D \end{cases} \tag{1}$$

where $\sigma(u) = Ae(u)$ is the stress tensor; A describes the Hooke's law; $e(u)$ is the strain tensor; f is the body force; t is the boundary traction force; n is the outward unit normal.

The optimization problem considered in the present study is the minimum compliance problem whose objective function is

$$J(\Omega) = \int_\Omega f \cdot u \,\mathrm{d}x + \int_{\Gamma_N} t \cdot u \,\mathrm{d}s \tag{2}$$

A constraint stating that the volume $V(\Omega)$ of material should not be bigger than a given upper bound $\overline{V}$ is

$$V(\Omega) = \int_\Omega \mathrm{d}x \leqslant \overline{V} \tag{3}$$

Another constraint stating that the cost $C(\Gamma_D)$ of support should not be bigger than a given upper boundary $\overline{C}$ is

$$C(\Gamma_D) = \int_{\Gamma_D} c(x)\,\mathrm{d}s \leqslant \overline{C} \tag{4}$$

where $c(x)$ is a fixed scalar field that describes the cost of support at point x. The function $c(x)$ means that the cost of support depends on the position. Finally, the set of admissible shapes is defined as

$$\mathcal{U}_{\mathrm{ad}} = \{\Omega \subset \mathcal{D},\ V(\Omega) \leqslant \overline{V},\ C(\Gamma_D) \leqslant \overline{C}\} \tag{5}$$

The optimization problem is defined as

$$\inf_{\Omega \in \mathcal{U}_{\mathrm{ad}}} J(\Omega) \tag{6}$$

The formulation given by Eq. (6) is ill–posed since an optimal structure can be further optimized by introducing infinitesimal holes. Such ill–posedness will manifest itself in the numerical results as the phenomenon of mesh-dependency, i.e., when the mesh of finite element and the grid of level set are refined, the optimal structure can be further optimized. Such an issue has been thoroughly investigated [28–30]. According to the results of [28,30], an effective way to regularize the optimization problem is to use a penalty of the perimeter of a structure, and this method is often used in the level set based shape and topology optimization [26,31]. Please refer to an excellent review of the level set method for structural topology optimization [32]. In our present work, the perimeter is added to the optimization problem as

$$\inf_{\Omega \in \mathcal{U}_{\mathrm{ad}}} J(\Omega) + \ell P(\Omega) \tag{7}$$