Comput. Methods Appl. Mech. Engrg. 272 (2014) 354–378

Contents lists available at ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Explicit feature control in structural topology optimization via level set method

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ARTICLE INFO

Article history:
Received 1 October 2013
Received in revised form 8 January 2014
Accepted 9 January 2014
Available online 20 January 2014

Keywords:
Topology optimization
Feature control
Level-set
Signed distance function

ABSTRACT

The present paper aims to address a long-standing and challenging problem in structural topology optimization: explicit feature control of the optimal topology. The basic idea is to resort to the level set solution framework and impose constraints on the extreme values of the signed distance level set function used for describing the topology of the structure. Numerical examples are also presented and discussed to illustrate the effectiveness of the proposed approach.

1. Introduction

Since the pioneering work of Bendsoe and Kikuchi [1], many efforts have been devoted to topology optimization of structures, which tries to place the available material within a prescribed design domain in an optimal way in order to achieve the best structural performance. Although first initialized in the field of mechanical design, topology optimization has found its application in a wide range of physical disciplines including acoustics, electromagnetics and optics nowadays. We refer the readers to [2–4] and the references therein for a state-of-the-art review of topology optimization.

Topology optimization of continuum structures, which is, in its mathematically nature, a discrete optimal control problem of the coefficients of partial differential equations in infinite dimensional space, is the most challenging structural optimization problem [5]. In order to solve topology optimization problems, many approaches have been proposed. In 1988, Bendsoe and Kikuchi developed the homogenization method, which is a natural extension of Cheng and Olhoff's work [6] on thickness optimization of thin solid elastic plates, for numerical topology optimization in their seminal paper [1]. Later on, density approach, where an artificial density field is introduced to represent material distribution and the intermediate values of density are penalized, was suggested in [7–9] by different authors. Because of its effectiveness and simplicity, density approach and its variants have found a lot of applications in optimal design of structures [5], and become the most popular approach in structural topology optimization. In around 2003, a new approach for structural topology optimization, i.e., level set approach, was suggested in [10,11]. In this approach, a level set function whose zero contour represents the boundary of the structure is introduced in the problem formulation and shape sensitivity is employed for its evolution. Because of its natural advantages for the solution of some specific topology optimization problems (e.g., topology optimization with boundary-independent loads/conditions, stress-related topology optimization), level set-based topology optimization

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http://dx.doi.org/10.1016/j.cma.2014.01.010
0045-7825/

approaches have received ever-increasing attention in recent years. At this position, it is worth noting that the main difference among these well-established topology optimization approaches consists in how to approximate the characteristic function used for indicating the topology of a structure and the corresponding numerical treatments.

One long standing problem in structural topology optimization, which can be traced back to the pioneering work of Cheng and Olhoff [6], is to regularize the corresponding problem formulation in order to get mesh-independent results. This is because the naive formulation of topology optimization often lacks solution and the corresponding design space is not closed [6,12–14]. In order to guarantee the existence of solutions and the convergence of optimization results with mesh refinement, many local or global regularization schemes are suggested in literature. We refer the readers to [14,15] for comprehensive reviews on this aspect. Another long standing problem in structural topology optimization, which is closely related to regularization, is feature control of optimal structural topology. The goal of feature control is to restrict the length scales appeared in the optimal structure (e.g., the minimum/maximum cross sectional area of the structural members, the minimum/maximum radius of the holes) and therefore make the resulting optimal designs more reliable and manufacturable. Another advantage of taking feature control into consideration is that it can help make topology optimization problems well-posed by preventing the outcome of "chattering" optimal designs (i.e., optimal design with microstructures) with infinitesimal sizes of structural features.

In order to achieve feature control in structural topology optimization, many approaches have been proposed. In [16], the author showed that filtering of the design sensitivities cannot only eliminate the undesirable checkerboard patterns and achieve mesh-independent results but also control the minimum size of structural features implicitly through the filter radius. Since then the filter approach has been employed by many researchers for numerical stabilization and feature control in different kinds of topology optimization problems. Although developed with the purpose of ensuring the existence of solutions, numerical experiments showed that the slope-constrained formulation of Petersson and Sigmund, where the density slope is constrained pointwisely, [17] also has the effect of feature control for optimal topology. Poulsen proposed a so-called MOLE (MOnotonicity based minimum LEngth scale) method for imposing minimum length scale in topology optimization [18]. In this approach a global constraint functional $L(\boldsymbol{\rho}, d)$, which measures the magnitude of variations of density field along specific directions within some prescribed length scale, is introduced to the problem formulation. It can be proved that feature control with minimum length scale d can be achieved by letting $L(\boldsymbol{\rho}, d) \leqslant 0$. Guest et al. developed a minimum length scale control approach for topology optimization by using nodal values of density field as primary design variables [19]. A projection operator is then employed to project the nodal values of density field onto the element space to determine the element density used for stiffness interpolation. The parameter $r_{\min}$, which is the size of the support set of the weight function appeared in the projection operator, determines the minimum allowable sizes of features in the optimal topology. Guest also suggested a scheme for imposing maximum length scale in topology optimization [20]. This is achieved by checking the satisfaction of an inequality at every point occupied by solid material. Accordingly, the radius of the circular test region $r_{\max}$ serves as the maximum allowable member sizes in the optimal topology. Feature control can also be achieved in level set-based method. Chen et al. proposed an elegant variational approach to realize feature control in topology optimization [21]. In this approach, a non-local quadratic energy functional, which describes the interaction between different points on structural boundary, is introduced to favor the formation of thin elongate structures with a fixed width. The same method has also been employed by Luo et al. to design hinge-free compliant mechanisms [22]. Besides the above methods, it is found that the wavelet-based [23,24], robust formulation based [25–28] and strain energy based [29] topology optimization approaches can also be used to control the structural feature sizes in an implicit way. Interestingly, how to include desired engineering features in the framework of optimal design was discussed in [30]. Furthermore, it is also worth noting that including total perimeter constraint in problem formulation can also control the minimum structural feature size indirectly as shown and discussed in [31–33].

Although remarkable achievements have been made for feature control in topology optimization, there is still room for further improvement of the existing methods. For example, the sensitivity filter and slope-constrained approaches suffer from the existence of grey elements along the boundary of the structure. Guest's projection scheme for minimum length scale control also has the same problem. Although almost 0–1 designs can be obtained by resorting to the so-called continuation approach, the corresponding computational efforts are not negligible. The common problem of Guest's maximum length scale control approach and Poulsen's minimum length scale control approach is that a large number of nonlinear constraints must be dealt with thereby increasing the computational expense. Chen et al.'s method seems tough for numerical implementation since a non-local energy term involving double layer boundary integral must be handled. As pointed in [15], the development of more efficient local and explicit feature control approaches is still very needed.

The present paper aims to develop an efficient, no post-processing/continuation, local and explicit scheme for complete control of the feature sizes in topology optimization which can, at the same time, generate pure 0–1 designs. The layout of the rest of the paper is as follows. In Section 2, the basic idea is explained in detail and the mathematical foundation is also laid for the proposed approach. A level-set based feature control formulation is then described in Section 3. Section 4 is devoted to the discussion of the corresponding numerical implementation aspects. The effectiveness of the proposed approach is demonstrated through several numerical examples in Section 5. Finally, some concluding remarks are provided in Section 6.

2. Mathematical foundation

Although feature control problem has been discussed intensively in the topology optimization literature, it is surprising to find that there are seldom discussions on quantitative and explicit definitions of some key terminologies involved in it, such