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A computational formulation for constrained solid and liquid membranes considering isogeometric finite elements

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ABSTRACT

A geometrically exact membrane formulation is presented that is based on curvilinear coordinates and isogeometric finite elements, and is suitable for both solid and liquid membranes. The curvilinear coordinate system is used to describe both the theory and the finite element equations of the membrane. In the latter case this avoids the use of local cartesian coordinates at the element level. Consequently, no transformation of derivatives is required. The formulation considers a split of the in-plane and out-of-plane membrane contributions, which allows the construction of a stable formulation for liquid membranes with constant surface tension. The proposed membrane formulation is general, and accounts for dead and live loading, as well as enclosed volume, area, and contact constraints. The new formulation is illustrated by several challenging examples, considering linear and quadratic Lagrange elements, as well as isogeometric elements based on quadratic NURBS and cubic T-splines. It is seen that the isogeometric elements are much more accurate than standard Lagrange elements. The gain is especially large for the liquid membrane formulation since it depends explicitly on the surface curvature.

1. Introduction

Membranes are computationally challenging structures. Their geometry can be complex, they may undergo large deformations, large rotations and large strains – and thereby behave highly nonlinear – and they are characterized by several physical instabilities: They are unstable in compression, unstable for out-of-plane loading (in the case of zero in-plane tension), unstable for pressure loading (in the case of rubber membranes) and unstable w.r.t. in-plane loading (in the case of liquid membranes). The aim of this paper is to formulate a general, 3D, geometrically exact and fully nonlinear computational membrane model that accounts for pressure loading as well as volume, area, and contact constraints and is suitable for both solid and liquid membranes. Our focus is on pure membranes, i.e. curved, surface structures that do not support in-plane compression, out-of-plane bending, and shear.[1] Such a restricted focus is useful due to the large range of membrane applications: they appear as inflatable and pressurized structures, like balloons, tubes and airbags; as fabrics, tents, canopies, parachutes, foils and sails; as water-filled membrane structures, like inflatable dams; as biological membranes, like blood vessels, cells, diaphragms, aneurysms and lung alveoli; as liquid droplets, menisci, bubbles, foams and sprays; as thin sheets and films – both liquid and solid – as atomistic membranes, like graphene sheets; as interacting membranes, e.g. adhering cells; and in the topic of form-finding and minimal surfaces.

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[1] We note that in the literature, the term membrane is often also used for the special case of 2D plane-stress structures.

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Computational formulations for 3D, nonlinear membrane go back to the seminal work of Oden ([31], see also [30]). Since then, the field has been continuously advanced, among others by Fried [15], Tang [45], Roddeman et al. [32], Contri and Schreer [10], Wriggers and Taylor [50], Ibrahimbegovic and Gruttmann [19], Haseganu and Steigmann [18], Gosling and Lewis [16], Muttin [28], Wu et al. [51], Bonet et al. [6], Rumpel and Schweizerhof [33], Stanuszek [43] and Weinberg and Neff [47]. Many of these works are concerned with the topic of wrinkling due to in-plane compression. Relevant to membranes is also the work on shells, since they typically contain membranes as special cases. Geometrically exact computational shell formulations go back to the work of Simo [41,42]. A review of the topic can be found in [4]. Recently, shell formulations have been proposed in the context of isogeometric analysis [22,3,29]. The fact that solid membranes can be extracted out of shell theory does not imply that this will work robustly in computations, which is why computational membrane formulations are important by themselves. Computational formulations based on curvilinear coordinates have been considered rigorously, both for membranes [1] and shells [2]. Another recent development are rotation-free shell formulations, as they have been considered by Flores and Estrada [14], Linhard et al. [25], and Dung and Wells [13] and recently Benson et al. [3] and Nguyen-Thanh et al. [29] for isogeometric analysis. Isogeometric formulations allow the formulation of C^1-continuous surface formulations that are advantageous for flow simulations [21] and sliding contact [11,46]; in this regard see also [34,35] for Hermite-based, C^1-continuous contact surfaces. Relevant to membranes is also the topic of live pressure loading [9,39]. Membranes are also an interesting subject in shape optimization [5,27].

This paper is concerned with several modeling aspects that are not considered in the computational formulations mentioned above. The most important of these are the inclusion of liquid membranes and the inclusion of several corresponding constraints and properties (like contact angles) into our formulation. We are also motivated by the provision of a novel pure membrane formulation for solids. This formulation is based on the theoretical framework of Steigmann [44], which has not yet been extended to a computational formulation. The presented formulation also contains several further merits and novelties: It allows for a split between in-plane and out-of-plane contributions, which is used to construct a new formulation for liquid membranes. It admits arbitrary elastic material models for solid and liquid membranes. In the case of liquid membranes, the formulation is capable of capturing the complex mechanical behavior at liquid interfaces that is a consequence of different interface energies and the surface topology.[2] The formulation constitutes an explicit formulation of the liquid surface that is more general than the explicit finite element formulation of Brown et al. [8], and is an alternative to immersed boundary and phase-field methods, e.g. see Dong [12] for a recent example. In distinction to the work of Javili and Steinmann [20], which extends surface energies to bulk systems, our formulation considers problems that are purely described by surface quantities.

Our formulation is based purely on displacement-based finite elements and can be used with any kind of such elements. It includes, in particular, isogeometric NURBS elements to capture the deforming surface geometry to high-accuracy, even for comparably coarse discretizations. It is straight forward to implement in an existing FE framework. It avoids the need to transform derivatives between configurations and avoids the use of local cartesian coordinate systems. Shell models are often formulated using local cartesian coordinate systems, as this allows using classical constitutive relations formulated in this manner [49]. To our mind, there is no need for such a detour: The balance laws, kinematics, constitutive relations as well as the FE weak forms and corresponding FE arrays can all be formulated efficiently in the curvilinear coordinate system. The capabilities of the presented formulation are demonstrated by several challenging computational examples, considering pressure loading, inflation and contact.

The following section presents the theory of nonlinear membranes in the framework of curvilinear coordinates, considering pressure loading, volume, area and contact constraints. Section 3 proposes a straight-forward finite element implementation of the theory, and Section 4 presents several examples of solid and liquid membranes to illustrate the capabilities of the present formulation.

2. Nonlinear membranes

In this section, we summarize the theory of nonlinear membranes in the framework of curvilinear coordinates. The membrane kinematics, constitution and balance laws in strong and weak form are discussed, and various kinds of constraints are considered. Details on curvilinear coordinates can for example be found in [24,44].

2.1. Surface description in curvilinear coordinates

The membrane surface, denoted $\mathcal{S}$, is fully characterized by the parametric description

$$\boldsymbol{x} = \boldsymbol{x}(\xi^1, \xi^2). \tag{1}$$

This corresponds to a mapping of the point (ξ^1, ξ^2) in the parameter domain $\mathcal{P}$ to the material point $\boldsymbol{x} \in \mathcal{S}$. In the following, Greek letters are used to denote the two indices 1 and 2. Summation is then implied on repeated indices. The tangent vectors to coordinate ξ^α at point $\boldsymbol{x} \in \mathcal{S}$ are given by

[2] The current formulation is restricted to quasi-static, continuum mechanical systems. Dynamical, atomistic and non-mechanical behavior are outside our present scope.