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An augmented-Lagrangian method for the phase-field approach for pressurized fractures

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ABSTRACT

In the modeling of pressurized fractures using phase-field approaches, the irreversibility of crack growth is enforced through an inequality constraint on the temporal derivative of the phase-field function. In comparison to the classical case in elasticity, the presence of the pressure requires the additional constraint and makes the problem much harder to analyze. After temporal discretization, this induces a minimization problem in each time step over a solution dependent admissible set. To avoid solving the resulting variational inequality corresponding to the first order necessary conditions, a penalization approach is used, commonly, to remove the inequality constraint. It is well-known that for large penalty parameters the algorithm suffers from numerical instabilities in the solution process. Consequently, to avoid such a drawback, we propose an augmented Lagrangian algorithm for the discrete in time and continuous in space phase-field problems. The final set of equations is solved in a decoupled fashion. The proposed method is substantiated with several benchmark and prototype tests in two and three dimensions.

1. Introduction

Presently, crack propagation in elastic and porous media is one of the major research topics in energy and environmental engineering. Consequently, many models and numerical techniques have been investigated so far. Specifically, Griffith's model [1] for quasi-static fracture evolution has been successfully applied. Here, the crack propagates if the rate of elastic energy decrease per unit surface area of the increment step is equal to the quasi-static critical energy release rate G_c. The crack does not move if the elastic energy release rate is less than G_c. On the contrary, it is unstable if G_c exceeds the critical rate. Griffith found that G_c is related to the crack surface energy increase. The solution of crack representation and propagation requires special techniques for their numerical treatment. In recent years, different approaches have been proposed such as the extended finite element method by Moes et al. [2] based on the partition of unity method of Babuska and Belenk [3] in which the displacement field is enriched with discontinuities. In addition, fixed-mesh approaches such as phase-field techniques have gained increased interest by studies from Francfort and Marigo [4], Bourdin et al. [5,6], Miehe et al. [7,8], Borden et al. [9], Hofacker and Miehe [10]. Instead of modeling the discontinuities explicitly (like in the extended finite element method), the lower-dimensional crack surface is approximated by a phase-field function. This introduces a diffusive

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transition zone (brittle zone or mushy-zone are also common expressions depending on the discipline) between the broken and the unbroken material; see Fig. 1.

The major advantages of using phase-field modeling for crack propagation are fourfold. First, it is a fixed-mesh approach in which remeshing is avoided. Second, the model is purely based on energy minimization and therefore, crack nucleation, propagation and the path are automatically determined (avoiding calculation of additional components such as stress intensity factors). Third, multiple joining and branching of cracks do not require any additional techniques. Consequently, phase-field modeling allows simple handling of large and complex fracture networks. Fourth, crack growth in heterogeneous media does not require any modification in the framework. Quantities of interest such as the crack opening displacement (the aperture) can be recovered with the help of the phase-field function. From the application point of view, we are specifically interested in pressurized fractures and their propagation, which are of particular interest in dam constructions, subsurface modeling, blood flow with damaged tissue, and oil recovery processes. In recent years, several methods for pressurized fracture and crack propagation have been proposed. These include an implicit moving mesh algorithms of Lecampion and Detournay [12]; moving-mesh approach with local grid refinement by Schrefler et al. [13]; a special zero-thickness finite element approach by Carrier and Granet [14]; the partition of unity and extended finite element approaches by Irzal et al. [15]; and finally, boundary element approaches by Ganis et al. [16] and Castonguay et al. [17]. To the best of our knowledge, the previously mentioned phase-field approach was first applied to pressurized cracks by Bourdin et al. [18], and a rigorous model investigation was first undertaken by Mikelić et al. [19,11] (See Fig. 2).

In this paper, we extend the previous studies [19,11]. Namely, the penalization of the irreversibility condition for crack growth is modified. It is well-known from Lootsma [20] and Murray [21] that simple penalization leads to numerical instabilities due to ill-conditioning of the constraint Hessian. Consequently, another (but computational costly) method is proposed by Mikelić et al. [11]. We circumvent these drawbacks in adapting a robust method from optimization: the augmented Lagrangian method dating back to Hestenes [22], Powell [23] and proposed by Fortin and Glowinski [24], Glowinski and Tallec [25] for use in discretized differential equations. In particular, we consider the augmented Lagrangian in a function space setting similar to Ito and Kunisch [26, Chapter 4].

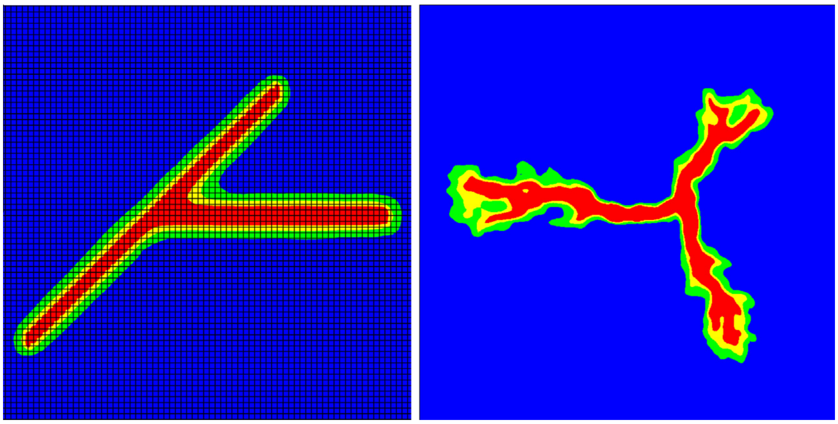

Fig. 1. Explication of the fixed-grid finite element phase-field approach: a (lower-dimensional) crack is approximated with the help of a phase-field function as shown in the left figure. The phase-field is an indicator function with values 0 in the crack (here in red) and 1 in the unbroken zone (here in blue). The mushy-zone provides a smooth interpolation between 0 and 1 indicated in yellow and green. On the right side, the major advantages for the phase-field approach are shown: joining, branching and nonplanar crack growth in possibly heterogeneous media (figure taken from [11]). (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this article.)

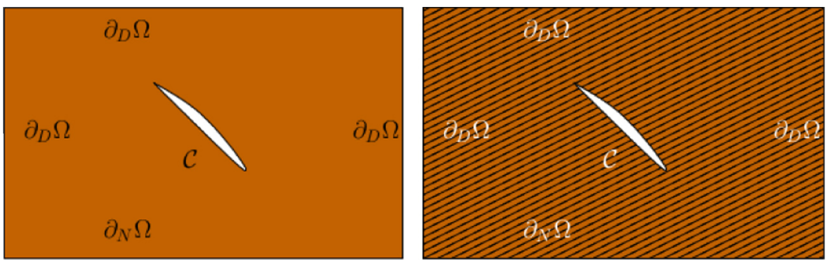

Fig. 2. Prototype configurations of the setting. The difference between Bourdin's approach [18] (left) and our approach (right) is that he works in an elastic medium whereas we work in a poroelastic setting.