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Constraints handling in Nash/Adjoint optimization methods for multi-objective aerodynamic design ☆

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ABSTRACT

Game theory and its particular Nash games are gaining importance in multi-objective optimization in engineering problems over the past decade. Among ongoing advances in optimization methods and tools, many applications including mathematical modeling with constraints in different areas remain challenges in industrial design environments. This paper describes a constraint handling algorithm to extend the use of Nash games to a more realistic multi-objective aerodynamic optimization when taking into account constraints. A competitive Nash game with constraints is adapted to a cooperative game, in which Nash procedure acts as a sub-game to calculate the equilibrium point and collaborates with a player in charge of constraints. The partial gradient of each objective instead of design variables is used as elitist information exchanged symmetrically in sub-game. Existence and equivalence of the solution are analyzed and proved based on Brouwer fixed point theorem. Numerical experiments on 2D multi-objective aerodynamic optimization with lift and/or geometry constraints are implemented, and results show that constraints are satisfied.

1. Introduction

Since John Nash published in 1950 his original contribution to non-cooperative game theory [1], introducing the fundamental concept of Nash equilibrium, the theory has been applied to numerous cases in various areas. Over the past decade, Nash equilibrium theory became an efficient tool to solve multi-objective optimization problems in aerodynamics [2,3] and other relative fields [4,5]. Among ongoing advances in optimization methods and tools, many applications including mathematical modeling with constraints in different areas remain challenges in industrial design environments. However, practical aerodynamic optimization problems involve constraints, such as drag reduction under fixed lift or lift to drag ratio maximization under fixed volume of a wing, etc. The computational design system will attempt to find the optimal solution satisfying the constraints.

The Nash equilibrium is the solution of a game based on symmetric information exchanged by Nash players. Equilibrium is reached when each player, constrained by the strategy of others, cannot improve further his own criterion. Constraints associated to objective function are implemented by partial design variables in Classical Nash Game (CNG). Thus symmetric elitist information exchange in the Nash game definitively changes the constraint value enforced by each player's decision. In

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this paper, authors propose a new perspective on constraints handling in Nash/Adjoint optimization methods for multi-objective aerodynamic design.

Literature survey

Many optimization problems in science and engineering involve a number of constraints which the optimal solution must satisfy. This section presents a brief review of literatures with focussing on how various multi-objective/criterion optimization methods deal with constraints' satisfaction. K. Deb [6] classified constraint handling methods used in classical optimization algorithms into two groups: (i) *generic* methods that do not exploit the mathematical structure (whether linear or nonlinear) of the constraint, and (ii) *specific* methods that are only used to decide if a search point is feasible or not.

Generic methods, such as the penalty function method [6–10] and the constraint aggregation method [12–15] as well as hybrid EAs for constrained optimization [16], are popular, because each one of them can be easily applied to any problem without important change in the algorithm. But since these methods are generic, the performance of these methods in most cases is not satisfactory.

Penalty functions decrease the fitness of captured infeasible solutions to prefer feasible solutions in the selection process. With Evolutionary Algorithms (EAs), the individual is penalized according to its constraint violation which is the sum of the violation of all constraints. A penalty term can be constructed based on constraint violation. An extended objective function is defined by introducing the penalty term into the original objective function, then to optimize the extended objective function. The penalty function approach involves a number of penalty parameters which must be set right in any problem to obtain feasible solutions. It is difficult to select an appropriate value for the penalty coefficient that adjusts the strength of the penalty. Taking into account the difficulty to determine the penalty parameter, this led researchers to devise sophisticated penalty function approaches such as multi-level penalty functions, dynamic penalty functions [7]. However, ideal control of the coefficient is problem dependent [11] and it is difficult to determine a general control scheme.

Farmani and Wright [7] proposed a two-stage adaptive fitness formulation method to ensure that slightly infeasible solutions with a low objective function value remain fitted. The main advantage of this method is that it does not require any parameter tuning. Huang et al. [9] proposed a co-evolutionary differential evolution for constrained optimization. Two populations are used in this method. The first population contains a set of penalty factors and is used to evolve decision solutions, while the second population consists of decision solutions and is employed to adapt penalty factors. These two populations evolve interactively and self-adaptively. K. Deb [6] proposed a pairwise comparison used in tournament selection in EAs which does not need any penalty parameter. In this method, when comparing pairwise individuals, (1) any feasible solution is preferred to any infeasible solution; (2) between two feasible solutions, the one with better objective function value is chosen; and (3) between two infeasible solutions, the one with smaller constraint violation is chosen. Runarsson and Yao [10] proposed a stochastic-rank-based approach. Takahama and Sakai [11] proposed the α constrained method. This method can convert an algorithm for unconstrained optimization problems into an algorithm for constrained optimization problems by the α level comparison [11].

Constraint aggregation transforms the Nonlinear Programming Problem (NLP) into a Multi-objective Optimization Problem (MOP) [13]. A constrained optimization problem is viewed as a constrained satisfaction problem. The main characteristic of this kind of methods are twofold: (1) converting the original constrained optimization problem into unconstrained multi-objective optimization problem and (2) exploiting multi-objective optimization techniques to solve the converted problem. Three mechanisms taken from multi-objective optimization are frequently incorporated into constraint-handling techniques: (1) using Pareto dominance as a selection criterion [14]; (2) using Pareto ranking to assign fitness in such a way that non dominated individuals are assigned a higher fitness value; and (3) splitting the population into sub-populations that are evaluated with respect to the objective function or with respect to a single constraint of the problem. The difficulty is that solving multi-objective optimization problems is a more difficult and expensive task than solving single objective optimization problems.

Hybrid methods are a combination of different algorithms and/or mechanisms. Y. Wang et al. [16] proposed a hybrid EAs and an adaptive constraint-handling technique for numerical and engineering constrained optimization problems. The hybrid EAs incorporates simplex crossover and two mutation operators (diversity mutation and improved Breeder Genetic Algorithms (BGAs) mutation) to generate the offspring population. In addition, the adaptive constraint handling technique consists of three main situations, i.e., infeasible situation, semi-feasible situation, and feasible situation. Only one situation is applied at each generation according to whether all the individuals are infeasible, there are feasible and infeasible individuals, or all the individuals are feasible. Meanwhile, one constraint-handling mechanism is designed according to the characteristic of the current situation.

Specific methods, such as the cutting plane method, the reduced gradient method, and the gradient projection method, are applicable either to problems having convex feasible regions only or to problems having a few variables, because of increased computational burden in specific methods with the large number of variables. However, generating initial feasible points is difficult and computationally demanding when the feasible region is very small. Especially if the problem has equality constraints, it is almost impossible to find initial feasible points. Special representations and operators method as well as repair algorithms are others specific methods. Special representations and operators are designed to represent only feasible