Comput. Methods Appl. Mech. Engrg. 271 (2014) 144–166

Contents lists available at ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Discontinuities without discontinuity: The Weakly-enforced Slip Method

G.J. van Zwieten [a,c,*], E.H. van Brummelen [a,b], K.G. van der Zee [a,d], M.A. Gutiérrez [c], R.F. Hanssen [e]

[a] Eindhoven University of Technology, Department of Mechanical Engineering, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[b] Eindhoven University of Technology, Department of Mathematics & Computer Science, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
[c] Delft University of Technology, Department of Mechanical, Maritime and Materials Engineering, P.O. Box 5058, 2600 GB Delft, The Netherlands
[d] University of Nottingham, School of Mathematical Sciences, University Park, Nottingham NG7 2RD, UK
[e] Delft University of Technology, Department of Civil Engineering and Geosciences, P.O. Box 5048, 2600 GA Delft, The Netherlands

ARTICLE INFO

Article history:
Received 29 April 2013
Received in revised form 17 October 2013
Accepted 11 December 2013
Available online 25 December 2013

Keywords:
Volterra dislocation
Finite element method
Weak imposition
Linear elasticity
Tectonophysics

ABSTRACT

Tectonic faults are commonly modelled as Volterra or Somigliana dislocations in an elastic medium. Various solution methods exist for this problem. However, the methods used in practice are often limiting, motivated by reasons of computational efficiency rather than geophysical accuracy. A typical geophysical application involves inverse problems for which many different fault configurations need to be examined, each adding to the computational load. In practice, this precludes conventional finite-element methods, which suffer a large computational overhead on account of geometric changes. This paper presents a new non-conforming finite-element method based on weak imposition of the displacement discontinuity. The weak imposition of the discontinuity enables the application of approximation spaces that are independent of the dislocation geometry, thus enabling optimal reuse of computational components. Such reuse of computational components renders finite-element modeling a viable option for inverse problems in geophysical applications. A detailed analysis of the approximation properties of the new formulation is provided. The analysis is supported by numerical experiments in 2D and 3D.

1. Introduction

The world is perpetually reminded of the fact that seismic hazard is still beyond reach of prediction. The difficulty does not just pertain to the exact moment of failure, which might never reach a practical level [1], but also to the nature of the risk and the extent to which stress accumulates. The main reason for this poor state of information is the lack of direct broad-scale measurements techniques to determine the magnitude and orientation of the stress tensor in the earth's crust. Information is obtained mainly from secondary observables, notably, by inferring stress evolution from accumulating changes due to earthquakes. Such inference proceeds from an understanding of tectonic mechanisms, the location and geometry of the section of the collapsed fault, and the direction and magnitude of fault slip. Such analyses are more and more based on local co-seismic surface displacements; information that has become available since the 1990s with the advent of space borne SAR and GPS measurements [2]. Analysis of this data has been rapidly adopted in recent years and is now routinely performed for all major earthquakes.

* Corresponding author at: Eindhoven University of Technology, Department of Mechanical Engineering, P.O. Box 513, 5600 MB Eindhoven, The Netherlands. Tel.: +31 641241437.

E-mail address: g.j.v.zwieten@tue.nl (G.J. van Zwieten).

0045-7825/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cma.2013.12.004

The predominant mechanical model to relate observations to physical processes is an elastic dislocation model, based on the assumption that on short time scales, nonlinear (plastic) effects are negligible. This model embeds a displacement discontinuity of given location and magnitude in an elastic medium, causing the entire medium to deform under the locked-in stress. Many different solution methods have been developed for this particular problem, based on analytical solutions or numerical approximations; see [3] for a survey of current methods. However, methods based on analytical solutions generally require severe simplifications, e.g., elastic homogeneity or generic geometries, which restrict their validity. The computational complexity of methods based on numerical approximation, on the other hand, is typically prohibitive in practical applications. Because in practice the surface displacements are given, and the dislocation parameters are unknown, the computational setting is always that of an inverse problem. Typical inversions require thousands of evaluations of the forward model, and computational efficiency is therefore imperative. The forward problems in the inversion process are essentially identical, except for the fault geometry, and reuse of computational components, such as approximate factors of the system matrix, can significantly enhance the efficiency of the inversion. Current numerical methods for seismic problems do however not offer such reuse options.

Finite-element methods are particularly versatile in terms of modeling capabilities in geophysics, allowing for elastic heterogeneity, anisotropy, and topography; things that cannot well be accounted for with currently used analytical and semi-analytical methods. In geophysical practice, finite-element methods are however often rejected for reasons of computational cost. The high computational cost can be traced to the condition that the geometry of the fault coincides with element edges to enable strong enforcement of the dislocation; see [4]. Consequently, the mesh geometry depends on the fault, which implies that mesh-dependent components such as the stiffness matrix and approximate factorizations of that matrix cannot be reused for different fault geometries and must be recomputed whenever the geometry of the fault changes. The recomputation of these components in each step of a nonlinear inversion process leads to a prohibitive overall computational complexity.

To overcome the complications of standard finite-element techniques in nonlinear inversion processes in tectonophysics, this paper introduces the *Weakly-enforced Slip Method* (WSM), a new numerical method in which displacement discontinuities are weakly imposed, in a similar manner as essential boundary conditions are enforced in Nitsche's method [5]. The weak enforcement of the discontinuity in WSM decouples the finite element mesh from the geometry of the fault, rendering the stiffness matrix and derived objects such as approximate factors independent of the fault and enabling reuse of these objects. The computational cost required for a single realisation of the fault geometry is comparable to that of standard FEM. However, reuse of components makes WSM significantly more efficient if many different fault geometries are considered. This makes finite-element computations based on WSM a viable option for nonlinear inverse problems.

A characteristic feature of WSM is that it employs standard continuous finite-element approximation spaces, as opposed to the conventional FEM split-node approach [4] which introduces actual discontinuities in the approximation space. Instead, WSM approximations feature a 'smeared' jump with sharply localized gradients. We will establish that the error in the WSM approximation converges only as $O(h^{1/2})$ in the $\boldsymbol{L}^2$-norm as the mesh width h tends to zero and that the error diverges as $O(h^{-1/2})$ in the energy norm. In addition, however, we will show that the WSM approximation displays optimal convergence in the energy norm away from the dislocation, i.e., optimal convergence rates are obtained on any subdomain excluding a neighborhood of the dislocation. Our numerical experiments convey that WSM also displays optimal local convergence in the $\boldsymbol{L}^2$-norm.

The Volterra dislocation problem bears resemblance to elliptic interface problems; see, for instance, [6–8] and the references therein. A fundamental difference is, however, that for the dislocation problem the solution itself is discontinuous, while for elliptic interface problems the solution generally only exhibits a discontinuity in the gradient. Due to the discontinuity, the solution to the dislocation problem does not reside in $\boldsymbol{H}^1(\Omega)$, as opposed to solutions to elliptic interface problems. Consequently, previous analyses for finite-element approximations for elliptic interface problem do not extend to the dislocation problem. The WSM formulation is similar to the Nitsche-type unfitted finite-element method for elliptic interface problems by Hansbo & Hansbo [6], and can in principle be obtained from the unfitted method by replacing the solution jump by the slip distribution. However, while the unfitted method in [6] is consistent and the applied broken approximation spaces are dense in $\boldsymbol{H}^1(\Omega)$, the WSM formulation is inconsistent and the applied continuous approximation spaces are not dense in $\boldsymbol{H}^1(\Omega \setminus \Gamma)$. As a result, approximation results for the unfitted method do not carry over to WSM. Let us also allude to the relation between the convergence results for elliptic interface problems away from the interface in [7] and the convergence results away from the dislocation in Section 4.3 below. However, despite the similarity of the results and some of the analysis techniques, e.g., extensions across the interface/dislocation, WSM contains some terms related to the discontinuity that require special treatment.

The remainder of this manuscript is organized as follows. Section 2 presents strong and weak formulations of Volterra's dislocation problem, and derives the corresponding lift-based finite-element formulation. Section 3 introduces the *Weakly-enforced Slip Method* based on two formal derivations, viz., by collapsing the support of the lift onto the fault and by applying Nitsche's variational principle. Section 4 examines the approximation properties of WSM. Section 5 verifies and illustrates the approximation properties on the basis of numerical experiments for several 2D and 3D test cases. In addition, to illustrate the generality of WSM, in the numerical experiments we consider several test cases that violate the conditions underlying the error estimates in Section 4, such as discontinuous slip distributions and rupturing dislocations. Section 6 presents concluding remarks.