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Post-buckling nonlinear static and dynamical analyses of uncertain cylindrical shells and experimental validation

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ABSTRACT

The paper presents a complete experimental validation of an advanced computational methodology adapted to the nonlinear post-buckling analysis of geometrically nonlinear structures in presence of uncertainty. A mean nonlinear reduced-order computational model is first obtained using an adapted projection basis. The stochastic nonlinear computational model is then constructed as a function of a scalar dispersion parameter, which has to be identified with respect to the nonlinear static experimental response of a very thin cylindrical shell submitted to a static shear load. The identified stochastic computational model is finally used for predicting the nonlinear dynamical post-buckling behavior of the structure submitted to a stochastic ground motion.

1. Introduction

The focus of this paper is on the post-buckling mechanical behavior of thin cylindrical shells, which is currently a subject of interest for which numerical predictions do not always match experimental measurements. This class of structures is commonly present in industrial equipments, such as silos, tanks for gas and liquid, reactor vessels, etc. The discrepancy between experimental observations and predictions is often due to the particular sensitivity of thin cylindrical shell structure to the presence of initial imperfections (heterogeneity of the materials, imperfect boundary conditions, inhomogeneous thickness induced by the manufacturing process and geometry). Note that for cylindrical shells of very small thickness, the geometrically nonlinear effects induced by large strains and large displacements must be taken into account. Numerous sensitivity analyses to standard geometric imperfections can be found in the literature, distinguishing several classes of external loads such as axial compression [1–3], pressure load [4,5] and shear load [6–10].

However, a generic sensitivity analysis of such structures with respect to any kind of imperfections requires the introduction of adapted non-deterministic approaches to represent uncertainties. For example, non-probabilistic approaches involving either interval analysis [11] or anti-optimization strategies [12] have been developed in the context of post-buckling analysis of structures. An experimental validation related to the identification of the buckling load of composite cylindrical shells can be found in [13].

Probabilistic approaches have also been used for representing the random uncertainties in the numerical computational models. The problems involving large nonlinear computational models, taking into account either or both the presence of random uncertainties and the stochastic nature of the loading requires appropriate strategies to properly achieve the

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dynamical analysis, see for instance [14,15]. More particularly, nonlinear stochastic buckling analyses have recently been conducted in which geometrical imperfections [16,17] and random boundary conditions [18] were modeled as Gaussian random fields whose statistical properties are issued from available experimental data. Non-Gaussian random fields have also been used for studying the sensitivity of buckling loads with respect to material and geometric imperfections of cylindrical shells [19]. Such probabilistic models of uncertainties will be referred to as parametric here as they focus the uncertainty only on specific aspects/parameters of the computational models selected by the analyst. It is also well known that the geometric imperfections particularly responsible for the sensitiveness of such structures [16,20].

In the investigation of unstiffened composite cylinders carried out in [17], the knowledge of existing missing composite fibers justified the modeling of material imperfections. Due to the lack of available experimental measurements, material imperfections were taken into account with chosen random properties. Despite the accurate modeling of the geometrical imperfections, the buckling load calculated with the nonlinear stochastic computational model was overestimated suggesting that other uncertainties, not considered in the analysis, were present and affected the experimental measurements.

An alternative approach, referred to as the nonparametric probabilistic approach, has been developed for situations in which the uncertainty cannot be singled out in one or a few parameters in the computational model. It allows the consideration of both system-parameter uncertainties and model uncertainties [21] by proceeding at the level of modal/reduced-order models developed on deterministic bases. Note that the nonparametric approach has been extended to uncertain nonlinear reduced-order models of geometrically nonlinear structures [22].

The development of such nonlinear reduced-order models requires first the selection of an appropriate deterministic basis for the representation of the response. This basis can be obtained by one of several techniques such as the Proper Orthogonal Decomposition method (POD method) [23–25], which is known to be particularly efficient for nonlinear static cases. One can also rely on linear elastic modes of vibration [26,27], or selected linear elastic modes appropriately enriched, e.g. [28], see [29] for a recent review. The parameters of the nonlinear reduced-order model of the mean structure can then be either deduced using the STEP procedure (which is based on the smart non-intrusive use of standard commercial finite element codes) [30,22,29] or from explicit construction as shown in [31] in the context of three-dimensional solid finite elements.

Having established the reduced-order model of the mean structure, uncertainties on the linear and on the nonlinear parts of the stiffness operator are introduced in the nonparametric framework. This is accomplished through the construction of a dedicated random operator with values in the set of all positive-definite symmetric real matrices whose mean value involves all linear, quadratic and cubic stiffness terms of the mean nonlinear reduced-order model [22]. The resulting stochastic nonlinear computational model is characterized by a single scalar dispersion parameter, quantifying the level of uncertainty in the stiffness properties which can easily be identified with experiments. Experimental validations based on this theory can be found in [31,32] for slender elastic bodies, e.g. beams.

The paper is organized as follows. Section 2 summarizes the main steps leading to the mean non-linear reduced-order computational model following the approach of [31]. Section 3 is devoted to the construction of the stochastic nonlinear computational model using the nonparametric probabilistic approach for modeling the random uncertainties. A Gaussian non-stationary second-order stochastic process is also introduced to represent the prescribed, earthquake-induced ground motions. An identification effort is carried out in Section 4 to calibrate the stiffness dispersion parameter of the stochastic nonlinear computational model from experimental measurements of the response of the cylindrical shell. Finally, the nonlinear post-buckling dynamical analysis of the uncertain cylindrical shell is carried out in Section 5 using the previously identified stochastic nonlinear computational model subjected to the prescribed ground motions.

2. Reduced-order computational model using 3D elasticity in large deformation

This section is devoted to the construction of a nonlinear reduced-order model in the context of elastodynamics involving geometrical nonlinearity.

2.1. Mathematical notations

From here on, the convention for summation over repeated indices will be adopted. Let $a(\mathbf{x},t)$ be a given function. The following notations are used: $a_{,j} = \partial a/\partial x_j,\ \dot{a} = \partial a/\partial t,\ \ddot{a} = \partial^2 a/\partial t^2$.

2.2. Description of the nonlinear boundary value problem

The structure under consideration is composed of a linear elastic material and is assumed to undergo large deformations inducing geometrical nonlinearities. A total Lagrangian formulation is chosen. Consequently, the dynamical equations are written with respect to the reference configuration. Let Ω be the three-dimensional bounded domain of the physical space $\mathbb{R}^3$ corresponding to the reference configuration taken as a natural state without prestress and subjected to the body force field $\mathbf{g}(\mathbf{x},t)$, in which $\mathbf{x}$ denotes the position of a given point belonging to domain Ω. The boundary $\partial\Omega$ is such that $\partial\Omega = \Gamma \cup \Sigma$ with $\Gamma \cap \Sigma = \emptyset$ and the external unit normal to boundary $\partial\Omega$ is denoted by $\mathbf{n}$. The boundary part Γ corresponds to the fixed part of the structure whereas the boundary part Σ is subjected to the external surface force field $\mathbf{G}(\mathbf{x},t)$. The displacement