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Artery active mechanical response: High order finite element implementation and investigation

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ABSTRACT

The active mechanical response of an artery wall resulting from the contraction of the smooth muscle cells (SMCs) is represented by a strain energy function (SEDF) that augments the passive SEDF recently reported in Yosibash and Priel [Z. Yosibash, E. Priel, *p*-FEMs for hyperelastic anisotropic nearly incompressible materials under finite deformations with applications to arteries simulation, Int. J. Numer. Methods Engrg., 88 (2011) 1152–1174]. The passive–active hyperelastic, anisotropic, nearly-incompressible problem is solved using high-order finite element methods (*p*-FEMs). A new iterative algorithm, named "*p*-prediction", is introduced that accelerates considerably the Newton–Raphson algorithm when combined with *p*-FEMs. Verification of the numerical implementation is conducted by comparison to problems with analytic solutions and the advantages of *p*-FEMs are demonstrated by considering both degrees of freedom and CPU.

The passive and active material parameters are fitted to bi-axial inflation–extension tests conducted on rabbit carotid arteries reported in Wagner and Humphrey [H.P. Wagner, J.D. Humphrey, Differential passive and active biaxial mechanical behavior of muscular and elastic arteries: basilar versus common carotid, J. Biomech. Engrg., 133 (2011) (Article number: 051009)]. Our study demonstrates that the proposed SEDF is capable of describing the coupled passive–active response as observed in experiments. Artery-like structures are thereafter investigated and the effect of the activation level on the stress and deformation are reported. The active contribution reduces overall stress levels across the artery thickness and along the artery inner boundary.

1. Introduction

Artery walls are anisotropic and nearly incompressible and consist of two main thin layers made of an elastin matrix embedded with stiff collagen fibers and smooth muscle cells (SMCs). In addition to the passive mechanical response due to the elastin and collagen fibers (well investigated in past studies), the SMCs contract in response to chemical stimulus thereby augment the passive response. Experiments show that the amount of tension generated by the SMCs is a function of the concentration of the chemical stimulus (dose–tension relation) and the amount of stretch exerted on the muscle fiber (tension–stretch relation) [1].

Artery walls are considered as being hyperelastic, thus a strain energy density function (SEDF) is sought which determines the constitutive equation (stress–strain relationship). Numerous studies propose different SEDFs for the passive mechanical response [2–4]. Some are phenomenological based, so the SEDFs are formulated to result in a stress–strain response that mimics the experimentally observed response [2], or semi-structural [5,6] in which some terms in the SEDF are related to the tissue microstructure. A fully-structural model, in which each component of the artery wall is modeled, individually, best describes the overall passive response, see e.g. the recent work by Hollander et al. [6]. However, fully structural models are very difficult to formulate because they require knowledge of arterial microstructure which is in most cases unavailable. Therefore, semi-structural models are preferred, and herein we modify the semi-structural *incompressible* hyperelastic SEDF by Holzapfel et al. [5] for the passive part of the artery-wall response.

The active response and its numerical treatment were scarcely addressed in past publications comparing to publications on the passive response. One of the early works on the subject is by Rachev and Hayashi [7] in which the SMCs contribution was considered by an additional term to the Cauchy stress in the circumferential direction. The magnitude of the added stress depends on the chemical concentration and the circumferential stretch ratio. There, no clear relation was provided between the concentration of the stimulating chemical and the developed active stress. The study showed that incorporation of SMCs resulted in a reduction of the circumferential Cauchy stresses. The "added stress" proposed in [7] was utilized by

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Masson et al. [8] to model the active response and to fit active material parameters from in vivo monitoring of the time dependent pressure response of a human carotid artery, assuming as in [7], that the SMCs fibers were circumferentially oriented. A different functional representation for the added active stress was used by Wagner and Humphrey [9] for simulation of inflation-extension experiments on the basilar and common carotid arteries of New-Zealand white rabbits. The functional representation for tension–dose relation was more specific, enabling the modeling of partial SMCs contraction. An incompressible one-layer cylindrical tube-like artery undergoing pure radial deformation (enabling an analytical solution to be obtained) was considered.

Another method of introducing the SMCs effect in the constitutive model is through an "active-SEDF", see e.g., [10,11]. The active-SEDF proposed in [10] does not incorporate the tension–dose relation and focuses on the passive, normal tone state (for which the pointwise activation level takes the form of Gaussian distribution function) and then on the maximum SMCs contraction. This results in a linear stress–strain relationship for maximal contracted SMCs thus it is only suited for the ascending part of the tension-stretch curve. The active-SEDF in [11] is based on a micro-mechanical approach so that the activation level is determined by a chemical kinetics model with an internal time-dependent variable, requiring the determination of many material properties. Schmitz and Bol [12] incorporated in the finite element (FE) framework an active-SEDF similar to the one in [11]. Uniaxial strip experiments on porcine medial strips reported by Herlihy and Murphy [13] under passive and active response were used to fit the active material parameters together with the collagen fiber dispersion reported by Dahl et al. [14] for the fit of the passive material parameters. Good agrement between the predicted and experimental results is reported, but the methods were not extended to artery-like structures and were restricted to strip specimens. It must also be noted that in [12] the implementation of the SEDF in the framework of FEs is not described and thus not verified.

In [15] the p-version of the FE method (p-FEM) was shown to perform very well for modeling the passive-response of artery-like structures and that slight compressibility which is mostly neglected in past studies was taken into consideration. Here we develop a new active-SEDF (aimed at augmenting the passive-SEDF in [15]) that is easily incorporated in the framework of p-FEMs. The use of p-FEMs based on the displacement formulation is motivated by the recent results [16–18,15] showing their advantages over conventional h-FEMs. p-FEMs were shown to be highly-efficiency for the analysis of isotropic hyperelastic materials and are locking-free for nearly incompressible Neo-Hook isotropic hyperelastic materials. These advantages in addition to the robustness of the p-FEM with respect to large aspect ratios and distortion of the elements, makes it especially attractive for modeling biological tissues as arteries. To the best of our knowledge this is the first study that uses p-FEMs to investigate the passive-active artery response. We present several "benchmark" problems used to verify our numerical implementation and demonstrate the superiority of p-FEMs over traditional h-FEMs in terms of degrees-of-freedom (DOFs) and CPU. A novel method, intrinsic to the hierarchic property of the p-shape functions, is exploited here to expedite the Newton–Raphson algorithm and dramatically reduce computational time. Following the verification of our methods we use experimental inflation–extension observations reported by Wagner and Humphrey [9] to fit the material parameters for the passive and active SEDFs.

In Section 2 the notations and the derivation of the active-SEDF are outlined and the ingredients required for implementation of the active model in the FE framework are explicitly presented. Three problems with analytic solution are utilized in Section 3 to verify our numerical implementation and to investigate the performance of the p- and h-FEMs. Fitting of passive and active model material parameters to experiments is outlined in Section 4. There we also investigate the p-FEM performance for a more realistic bilayer artery-like structure. In Section 5 we emphasize the effect of the various active parameters on the artery wall. We summarize our work and draw several conclusions in Section 6.

2. Notations and implementation of an active-SEDF in the framework of FEMs

The point of departure is a brief description of our notations for a fiber reinforced hyper-elastic material. The basic quantity is the deformation gradient $\mathbf{F} = \mathrm{Grad}\, \mathbf{x}(\mathbf{X},t) = \partial x^k(X^1,X^2,X^3,t)/\partial X^K\, \mathbf{g}_i \otimes \mathbf{G}^K$, where $\mathbf{x}(\mathbf{X},t)$ defines the placement of the point $\mathbf{X}$ at time t. $X^K,\ k=1,2,3$, are the material (reference) coordinates, $\mathbf{g}_i$ are the tangent and $\mathbf{G}^K$ the gradient vectors in the current and the reference configuration. Customary, the displacement vector $\mathbf{U}(\mathbf{X},t) \stackrel{\mathrm{def}}{=} (U_X,U_Y,U_Z)^T$ is introduced, i.e. $\mathbf{x} = \mathbf{X} + \mathbf{U}(\mathbf{X},t)$, and with this notation $\mathbf{F} = \mathbf{I} + \mathrm{Grad}\mathbf{U}(\mathbf{X},t)$. We interchange X^1, X^2, X^3 with X, Y, Z when appropriate. A general strain–energy density function (SEDF) for an isotropic hyperelastic material with two families of fibers used to model the passive response is denoted by, $\psi_{passive}(\mathbf{C},\hat{\mathbf{M}}_0,\hat{\mathbf{M}}_1) = \Psi_{passive}(\mathrm{I}_\mathbf{C},\mathrm{II}_\mathbf{C},\mathrm{III}_\mathbf{C},\mathrm{IV}_\mathbf{C},\mathrm{VI}_\mathbf{C})$, following Holzapfel et al. [5]. It depends on the invariants of the right Cauchy–Green tensor $\mathbf{C} = \mathbf{F}^T\mathbf{F} = (\mathbf{I}+\mathrm{Grad}\mathbf{U})^T(\mathbf{I}+\mathrm{Grad}\mathbf{U})$, and two unit direction vectors along collagen fiber directions $\hat{\mathbf{M}}_0$, and $\hat{\mathbf{M}}_1$. For example, using the Cartesian coordinate system in Fig. 1, the fibers directions are $\hat{\mathbf{M}}_0 = (\sin\beta, -\cos\beta\frac{Y}{\sqrt{Y^2+Z^2}}, \cos\beta\frac{Z}{\sqrt{Y^2+Z^2}})^T$, $\hat{\mathbf{M}}_1 = (-\sin\beta, -\cos\beta\frac{Y}{\sqrt{Y^2+Z^2}}, \cos\beta\frac{Z}{\sqrt{Y^2+Z^2}})^T$. The invariants of the Cauchy–Green tensor are

$$\mathrm{I}_\mathbf{C} = tr\mathbf{C}, \quad \mathrm{II}_\mathbf{C} = \frac{1}{2}((tr\mathbf{C})^2 - tr\mathbf{C}^2), \quad \mathrm{III}_\mathbf{C} = \det\mathbf{C} = (\det\mathbf{F})^2 \stackrel{\mathrm{def}}{=} J^2, \quad (1)$$

where $tr\mathbf{C}$ symbolizes the trace operator and the invariants that represent stretch in the fiber directions are

$$\mathrm{IV}_\mathbf{C} = \hat{\mathbf{M}}_0 \cdot \mathbf{C} \cdot \hat{\mathbf{M}}_0, \quad \mathrm{VI}_\mathbf{C} = \hat{\mathbf{M}}_1 \cdot \mathbf{C} \cdot \hat{\mathbf{M}}_1. \quad (2)$$

Following [5] we consider a strain–energy density function composed of three parts for modeling the passive response, an isochoric isotropic and a volumetric isotropic Neo-Hook parts representing the elastic matrix, and a transversely isotropic part representing the collagen fibers in the artery wall

$$\Psi_{passive}(\mathrm{I}_\mathbf{C},\mathrm{III}_\mathbf{C},\mathrm{IV}_\mathbf{C},\mathrm{VI}_\mathbf{C}) = [\Psi_{isoch}(\mathrm{I}_\mathbf{C},\mathrm{III}_\mathbf{C}) + \Psi_{vol}(\mathrm{III}_\mathbf{C})] + \Psi_{fibers}(\mathrm{IV}_\mathbf{C},\mathrm{VI}_\mathbf{C}), \quad (3)$$

The isochoric isotropic and volumetric isotropic parts are represented by a nearly incompressible Neo-Hookean SEDF:

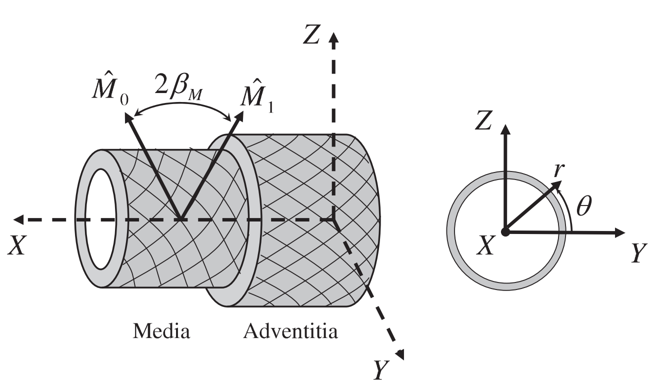

Fig. 1. Coordinate system in a typical artery.