Comput. Methods Appl. Mech. Engrg. 237–240 (2012) 192–211

Contents lists available at SciVerse ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

A topology optimization method based on the level set method for the design of negative permeability dielectric metamaterials

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ARTICLE INFO

Article history:
Received 4 November 2011
Received in revised form 29 April 2012
Accepted 30 April 2012
Available online 23 May 2012

Keywords:
Dielectric metamaterials
Negative permeability
Topology optimization
Finite Element Method
Level set method
Adjoint Variable Method

ABSTRACT

This paper presents a level set-based topology optimization method for the design of negative permeability dielectric metamaterials. Metamaterials are artificial materials that display extraordinary physical properties that are unavailable with natural materials. The aim of the formulated optimization problem is to find optimized layouts of a dielectric material that achieve negative permeability. The presence of grayscale areas in the optimized configurations critically affects the performance of metamaterials, positively as well as negatively, but configurations that contain grayscale areas are highly impractical from an engineering and manufacturing point of view. Therefore, a topology optimization method that can obtain clear optimized configurations is desirable. Here, a level set-based topology optimization method incorporating a fictitious interface energy is applied to a negative permeability dielectric metamaterial design problem. The optimization algorithm uses the Finite Element Method (FEM) for solving the equilibrium and adjoint equations, and design problems are formulated for both two- and three-dimensional cases. First, the level set-based topology optimization method is explained, and the optimization problems for the design of metamaterials are then discussed. Several optimum design examples for the design of dielectric metamaterials that demonstrate negative effective permeability at prescribed frequencies are provided to confirm the utility and validity of the presented method.

1. Introduction

This paper discusses a level set-based topology optimization method for the design of dielectric metamaterials that achieve a negative permeability at desired frequencies. Electromagnetic metamaterials are artificial materials that exhibit extraordinary electromagnetic properties not available in nature, such as a negative refractive index, that is, negative permittivity and permeability. The existence of such materials was first proposed by Veselago [1] in 1968. After Pendry et al. [2,3] and Smith et al. [4] showed that arrangements of split-ring resonators that have negative permeability and metallic wires that have negative permittivity can exhibit negative refraction at a certain frequency, considerable research was carried out to investigate the unusual properties of such materials, and develop certain applications, such as cloaking devices [6], waveguides [5], super lenses [7], leaky wave antennas [8], energy harvesting devices [9], and the like. Furthermore, recently, new types of metamaterials that utilize the magnetic and electric resonance phenomena of dielectric materials rather than effects primarily derived from metallic inclusions have been proposed [10–14]. These new dielectric metamaterials are expected to offer advantages due to improved manufacturability and the possibility of achieving isotropic metamaterials that provide advanced functions under no metallic loss.

Holloway et al. [11] showed theoretically that negative effective permittivity and negative effective permeability can be simultaneously achieved with appropriately designed dielectric spheres embedded in a host material. Subsequently, more practical structures based on this approach were suggested, such as structures using dielectric particles of two different radii [12], structures using identically sized spheres but with different values of dielectric constant [13], and arrays of cylindrical dielectric materials [14]. Experimental verifications are provided for three-dimensionally isotropic dielectric metamaterials consisting of an array of dielectric cubes that exhibit negative permeability [15], and an array of dielectric rods [16], and cubic dielectric particles [17] that exhibit negative permittivity and negative permeability simultaneously. Furthermore, applications such as all-dielectric cloaking devices [18,19], as well as waveguides [20] and leaky wave antennas [21] composed of dielectric materials and metallic plates, have been discussed.

Most electromagnetic metamaterials consist of periodic arrays of unit cells that are adequately small compared to the wavelength

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http://dx.doi.org/10.1016/j.cma.2012.04.022

of the target frequency, with cells composed of a layer of dielectric material, with or without metallic inclusions. The overall structure of such periodic arrays can be considered as an effectively homogeneous electromagnetic structure, so the electromagnetic metamaterial behaves as a material having negative properties exhibited globally, whereas the individual cell materials do not exhibit these properties. Several methods have been proposed to obtain the effective properties of electromagnetic metamaterials, such as homogenization methods based on the asymptotic-expansion approach [22–25] and the energy-based approach [26] that can be applied when the periodic unit cell can be considered as infinitely small compared to the wavelength, a method that computes the effective properties by averaging electric and magnetic fields in the unit cell [27], and methods that extract effective properties from S-parameters, namely, the complex transmission and reflection coefficients [28–31]. Such effective medium theories provide the basis for the design of metamaterial unit cells that can be used to construct useful metamaterials.

Several unit cell layouts have been proposed that achieve good performance at certain desired frequencies [32]. However, the unit cell layout crucially affects the performance of metamaterials, and it is usually difficult or time-consuming to find appropriate unit cell designs by trial and error methods, even for expert engineers. Thus, there is a need for systematic design methods that assist or simplify the design of effective metamaterials. One systematic approach for obtaining desirable unit cell designs is to apply a structural optimization method. Zhou et al. [33] proposed a level set-based structural optimization method for the design of double negative metamaterials, that is, metamaterials with negative permittivity and negative permeability. The aim of the optimization problem in this case was to find an optimized layout of metallic inclusions, and the objective function was formulated using current flow, instead of using the effective permittivity or permeability directly. Subsequently, Zhou et al. [34] proposed a level set-based structural optimization method in which the effective permeability is directly used as an objective function. Concerning the design optimization of metamaterial applications, Yamasaki et al. [35] proposed a level set-based structural optimization method for the design of composite right- and left-handed transmission lines consisting of a metallic waveguide with dielectric inclusions. The aim of optimization problem was to find the optimized configuration of the dielectric inclusion within the unit cell of the transmission line that provides desired dispersion behavior for the composite right- and left-handed transmission line. In level set-based structural optimization methods [36,37], the structural boundaries are represented by the iso-surface of a scalar function called the level set function, and the boundaries are evolved by updating the level set function using a Hamilton–Jacobi equation. However, this method is based on the concept of shape optimization, and only the boundaries of the target structure are changed during the optimization procedure, so topological changes such as the introduction of holes is not allowed, although the number of holes can be decreased during optimization.

On the other hand, topology optimization methods, the most flexible type of structural optimization method, allow not only changes in shape, but also topological changes that include increasing the number of holes in the design domain [38,39]. Such methods have been applied to a variety of problems, such as stiffness maximization problems [40], eigen-frequency problems [41], electromagnetic problems [42] and, recently, electromagnetic metamaterial problems. Diaz and Sigmund [43] proposed a topology optimization method for the design of negative permeability metamaterials using an S-parameter retrieval method, where the imaginary part of the effective permeability was minimized at a specific frequency, and several designs for metallic structures attached to dielectric substrates that achieved negative permeability were provided. Sigmund [44] proposed a topology optimization method for dielectric metamaterials to obtain dielectric material designs that minimize the effective permeability at a specific frequency, which also employed the S-parameter retrieval method to obtain the effective properties. Choi and Yoo [45] introduced the inverse homogenization method [46] for the design of magnetic materials that demonstrate a desirable prescribed effective permeability value. Zhou et al. [25] proposed an inverse homogenization method for the design of metamaterials, where both permittivity and permeability are simultaneously maximized. El-Kahlout and Kiziltas [47], and Otomori et al. [48] introduced inverse homogenization methods for the design of dielectric materials that demonstrate a desirable prescribed effective permittivity value, using an asymptotic expansion-based homogenization method and Genetic Algorithms (GAs) [47], and an energy-based homogenization method and density-based topology optimization [48], respectively. GAs have also been used to find optimized layouts of a metallic inclusion in the metamaterial unit cell for a negative permeability design problem [49], and for different multi-objective problems [50,51] in which the refractive index and impedance were simultaneously designed [50], and the bandwidth of the negative refractive index was maximized and the dissipation minimized [51].

The basic ideas of topology optimization are (1) the extension of the design domain to a fixed design domain and (2) the replacement of the optimization problem with material distribution problem in the fixed design domain using the characteristic function [52]. Since the characteristic function is a discontinuous function that represents the structure using a value of 0 or 1, the optimization problem is typically an ill-posed problem. To overcome this difficulty, the Homogenization Design Method (HDM) [38] and density approaches such as the SIMP method [39] have been proposed, where optimized configurations are represented as density distributions, with the density assuming continuous values from 0 to 1. However, such optimized configurations often include grayscale areas where the density is an intermediate value between 0 and 1. Although configurations including grayscale areas can be considered as composite materials, they are typically impractical to manufacture or meaningless in an engineering sense. To overcome this problem, several kinds of filtering scheme [53–58] have been proposed and applied to many problems, to provide optimized configurations that are free from grayscales.

Level set-based topology optimization methods that inherently obtain clear optimized configurations have also been proposed [59,60]. In these methods, the structural boundaries are implicitly represented by the iso-surface of the level set function, so grayscale areas do not appear. Yamada et al. [60] proposed a level set-based topology optimization method where the optimization problem is regularized using the Tikhonov regularization method, and the level set function is updated based on a reaction–diffusion equation. This method not only allows topological changes such as the introduction of holes during the optimization procedure, but also enables the complexity of optimized configurations to be controlled by using appropriate magnitudes of a regularization parameter. The method has been applied to many problems, such as stiffness maximization problems, eigen-frequency problems, compliant mechanism design problems [60], and problems to maximize thermal diffusivity [61], but application to dielectric metamaterial design problems has not yet been reported.

In this paper, the above-mentioned level set-based topology optimization method [60] is applied to the design of negative permeability dielectric metamaterials. The rest of this paper is as follows. Section 2 describes the formulation of the optimization problem for the design of dielectric metamaterials, for both two- and three-dimensional cases, and discusses the level set-based topology optimization method. Section 3 describes the