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A two-level time step technique for the partitioned solution of one-dimensional arterial networks

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ABSTRACT

In this work a numerical strategy to address the solution of the blood flow in one-dimensional arterial networks through a topology-based decomposition is presented. Such decomposition results in the local analysis of the blood flow in simple arterial segments. Hence, iterative methods are used to perform the strong coupling among the segments, which communicate through non-overlapping interfaces. Specifically, two approaches are considered to solve the associated nonlinear interface problem: (i) the Newton method and (ii) the Broyden method. Moreover, since the modeling of blood flow in compliant vessels is tackled using explicit finite element methods, we formulate the coupling problem using a two-level time stepping technique. A local (inner) time step is used to solve the local problems in single arteries, meeting thus local stability conditions, while a global (outer) time step is employed to enforce the continuity of physical quantities of interest among the one-dimensional segments. Several examples of application are presented. Firstly a study about spurious reflections produced at interfaces as a consequence of the two-level time stepping technique is carried out. Secondly, the application of the methodologies to physiological scenarios is presented, specifically addressing the solution of the blood flow in a model of the entire arterial network. The effects of non-uniformities of the material properties, of the variation of the radius, and of viscoelasticity are taken into account in the model and in the (local) numerical scheme; they are quantified and commented in the arterial network simulation.

1. Introduction

Numerical simulations of the cardiovascular system using a collection of simple distributed one-dimensional (1-D), or even lumped zero-dimensional (0-D), models have proven to be able to provide useful information under physiological and pathophysiological conditions. They give insight about the main characteristics of the flow and about the interplay among physical phenomena taking place in the systemic arteries [1–9].

From the computational viewpoint, there are some situations in which it is convenient to split the solution process into simpler problems, for instance: (i) in cases where the available computational implementations are based on black-box codes which cannot be accessed by the user [6], or (ii) in cases in which the number of arteries grows significantly, and the computational cost increases substantially [10]. Those situations pose the problem of developing decomposition strategies to deal with the coupled problem in an iterative manner. This implies resorting to numerical methods for solving the resulting nonlinear equations corresponding to the continuity equations considered at the interfaces among arterial segments.

Several approaches can be used for the solution of the blood flow in the entire arterial network. A first one corresponds to solve at each time step a trivial system of equations with information coming from the previous time step. This approach has drawbacks concerning stability due to continuity conditions at branching sites. A second option is to solve at each time step an algebraic system of equations in which all the unknowns are properly coupled. Unlike the first approach, this is stable but may be expensive in view of the large level of coupling among degrees of freedom within arterial segments. To overcome these issues, in this work we use a combination of both methods to solve the stability issues of the first case still making possible to perform explicit computations within each arterial segment due to the implicit coupling at the interfaces among them. This mixed approach is achieved by planning a numerical method that makes use of a two-level time discretization as will be explained afterwards.

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The decomposition of an arterial network into subdomains can be performed in different ways. A first option is to split the entire cardiovascular system into subregions corresponding to specific vascular areas, for instance legs, arms, head, organs, and so on, see e.g. [11]. A second option, followed in the present work, is to explore the system into the constituent arterial segments as done in [12].

Each arterial segment is represented by the 1-D Fluid–Structure Interaction (FSI) model developed in [6] and is fed with proper boundary conditions at both segment boundaries. The discretization is carried out using a Taylor–Galerkin approach which yields an explicit scheme to solve for the volumetric flow rate, the pressure, and the lumen area for a single pipe. Moreover, in the present work the numerical scheme presented in [6] is further extended to deal with material and geometrical spatial inhomogeneities. In addition, the compatibility conditions that arise at the discrete level to close the discrete problem are adapted to this more general situation.

Continuity conditions are imposed among arterial segments, leading to a coupled network of deformable vessels. We propose to solve iteratively this coupled problem following the ideas developed in [13] for linear problems and recently extended in [14,15] to flow problems in rigid pipes and in [11] to hemodynamics. Previous developments of iterative techniques to couple iteratively 1-D FSI models with Taylor–Galerkin explicit numerical formulations can be found in [12]. There, the authors use relaxed Gauss–Seidel iterations that relies on a hierarchy among the local models, dictating a compatible choice of the coupling conditions (volumetric flow rate versus average pressure). The poor convergence properties and lack of flexibility in setting boundary conditions at the arterial segment interfaces of such method can create difficulties in real situations in which we need to couple hundreds of arterial segments.

In view of these problems, we resort to two alternatives to solve the nonlinear coupling problem: (i) the Newton method and (ii) the Broyden method. The performance of these methods is assessed through several examples of application.

Another contribution of the present work is the development of a two-level time step technique to increase computational efficiency. Firstly, we recognize an inner local time step; it is used to solve the blood flow dynamics at each arterial segment and it is in general determined by the Courant–Friedrichs–Lewy (CFL) condition. Secondly, we consider an outer global time step which is used to match the physical quantities of interest at interfaces among arterial segments. In other words, the global time step is the one responsible for enforcing the strong coupling among the segments. This is convenient also when coupling 1-D and three-dimensional (3-D) FSI problems to reduce the number of solutions of expensive 3-D problems with very small time steps (needed by time discretization strategy of the 1-D problem).

The generation of spurious reflections at interfaces between segments as a consequence of the proposed two-level time step technique is analyzed. In this regard we unveil the dependence of these reflections upon the wavelength and upon the ratio between the local and global time steps. In addition, we propose a simple interpolation scheme to reduce the reflections at interfaces whenever the local time step is different from the global time step.

Another aspect of the formulation, specific to the Newton method, is related to the approximation of the Jacobian in the presence of the two-level time step technique. Two procedures are compared which lead to different ways of approximating the Jacobian, and therefore to different versions of the (actually inexact) Newton method: (i) Jacobian computed using a finite difference scheme and (ii) Jacobian computed using an approximated tangent problem formulation.

Finally, to include the viscoelastic properties of the vessel wall in the 1-D model (see, e.g., [6,16–18]) we propose a split time advancing scheme. To consider non-uniformities of the material properties and a the variation of the radius [17] we have added some additional terms in the original model. Indeed, these have non-negligible effects, as we show in the results.

All the developments in the present contribution are mainly envisaged for the hemodynamics field. Therefore, besides the study of spurious reflections carried out in academic situations, the solution of an entire arterial tree model is presented to show the robustness of the strategy in a far more complex system.

This work is organized as follows. Section 2 presents the governing equations, in continuous and discrete forms, for the 1-D FSI model of a single arterial segment. Section 3 presents the global arterial network problem and corresponding coupling equations, together with the proposed iterative methods. Section 4 deals with academic applications focusing on iterations numbers and spurious reflections, while Section 5 presents a series of comparisons of the performance of the different developed alternatives in physiological scenarios. Section 6 closes this work with the final remarks.

2. 1-D FSI model equations

The 1-D FSI model provides a simplified representation of the blood flow in deformable vessels. Although incapable to give a detailed description of the full 3-D structure of the flow field (such as recirculation or wall shear stress), it can effectively describe the wave propagation phenomena due to the compliance of the wall. In this section we first describe the governing equations for the 1-D FSI model. Then we introduce a numerical discretization of the problem. Finally, we close the resulting discrete formulation with an appropriate set of compatibility conditions.

2.1. Mass and momentum conservation laws

The 1-D FSI model is derived from the incompressible Navier–Stokes equations, by making some simplifying assumptions and integrating over the cross-section of the artery $S(t,z)$, being $t \in (0,T]$ the time and $z \in [0,L]$ the axial coordinate, with L the length of the vessel (see Fig. 1). The pressure on each transversal section is assumed to be constant, and the axial velocity profile $s(r)$ is chosen a priori through the power-law relation $s(r) = \theta^{-1}(\theta+2)(1-r^{\theta})$, where r is the relative radial coordinate and θ is a proper coefficient. This is a commonly accepted approximation (see, for instance, [19,20]), where $\theta = 2$ leads to a parabolic velocity profile, while $\theta = 9$ leads to a more physiological profile, following the Womersley theory.

The resulting state variables are

$$A(t,z) = \int_{S(t,z)} \mathrm{d}S,$$

$$Q(t,z) = \int_{S(t,z)} u_{zF}(t,z)\mathrm{d}S,$$

$$P(t,z) = \frac{1}{A(t,z)} \int_{S(t,z)} p_F(t,z)\mathrm{d}S,$$

where A is the cross-sectional area, Q the volumetric flow rate, u_{zF} the fluid axial velocity, and P the average pressure. A straightforward derivation of the 1-D FSI model can be found in [21]. The resulting governing equations for continuity of mass and momentum are