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Stiffness and strength of tridimensional periodic lattices

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ABSTRACT

This paper presents a method for the linear analysis of the stiffness and strength of open and closed cell lattices with arbitrary topology. The method hinges on a multiscale approach that separates the analysis of the lattice in two scales. At the macroscopic level, the lattice is considered as a uniform material; at the microscopic scale, on the other hand, the cell microstructure is modelled in detail by means of an in-house finite element solver. The method allows determine the macroscopic stiffness, the internal forces in the edges and walls of the lattice, as well as the global periodic buckling loads, along with their buckling modes. Four cube-based lattices and nine cell topologies derived by Archimedean polyhedra are studied. Several of them are characterized here for the first time with a particular attention on the role that the cell wall plays on the stiffness and strength properties. The method, automated in a computational routine, has been used to develop material property charts that help to gain insight into the performance of the lattices under investigation.

1. Introduction

Lattices are regular structures obtained by tessellating a unit cell along independent periodic vectors. A lattice can be tailored by design to obtain unprecedented mechanical properties and access unexplored areas of the material property space [4,3,14,2]. Recent manufacturing techniques enable to build complex lattice components at the micrometer length scale with high accuracy, acceptable costs [7,29], and from a variety of solid materials, including metal alloys, polymers, glass and silicon rubbers.

The microscopic characteristics of a lattice component govern its macroscopic behaviour. When the scale of the component is significantly larger than the scale of the lattice, a direct approach involving the modelling of each cell is impractical. This strategy would result in considerably large models, which are likely unfeasible to manage. On the other hand, an appropriate alternative is the substitution of the discrete model with an equivalent continuum [25]. At the cost of loosing minor local details, this approach permits a substantial reduction of the computational effort, while still providing high accurate results.

In literature, there exist several methods to model the macroscopic properties of lattice materials. A force-based approach has been often applied to the unit cell of a lattice subjected to a multiaxial load. The elastic constants of the lattice have then been determined by solving each equilibrium problem individually. For example, Gibson et al. [15] obtained a first order estimate of the in-plane stiffness of hexagonal honeycombs, by assuming the lattice edges behave as Bernoulli beams. Warren and Kraynik [37] examined hexagonal lattices with non-constant edge thickness and modelled lattice edges as continua under plane stress. Zhu et al. [38] and Gong et al. [18] expressed the Young's modulus and the Poisson's ratio of open cell tetrakaidecahedral foams as a function of the relative density. For this cell topology, Ohno et al. [26] derived the buckling and yielding strength under uniaxial compression. Among other cell topologies, pin-jointed fully triangulated lattices have attracted a remarkable interest for their excellent structural properties. Deshpande et al. [10] studied in detail the performance of the regular-octet lattice, and derived the lattice stiffness, and the von Mises surfaces for buckling and yielding. Elsayed and Pasini [13] applied shape transformers [27] to study the effect of the edge cross section on the stiffness and strength of columns made out of octet lattices. Wallach and Gibson [35] analysed a lattice based on tetrahedral units, and evaluated the effect of the cell aspect ratios on the stiffness, and on the buckling strength of alternative lattice. Wang and McDowell [36] calculated the in-plane stiffness and strength of a selected bidimensional cell topologies, with respect to the onset of plastic yielding.

Other works proposed a more general analysis of the mechanics of lattices based on topology optimization. Bendsoe and Kikuchi [6], and later Hassani and Hinton [19], for example, derived a constitutive model for porous materials considering an elementary unit cell of size ϵ, with $\epsilon \to 0$. The virtual work principle was first applied to determine the deformation energy of the unit cell. The effective stiffness of the porous material was then obtained as the average, over the unit cell volume, of the stiffness of the solid material, weighted by the unit cell deformation energy.

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The interest in vibration reduction and bandgaps has motivated the search for other approaches [23,28]. The following can be loosely specified as procedural steps of these methods. (i) A displacement model is first proposed to approximate the displacements of the continuous medium within the unit cell; (ii) physical quantities of interest, such as natural frequency, stiffness, and strain energy, are calculated for the discrete lattice; (iii) the parameters of the continuous model are then determined by equating the relevant quantities of the discrete to those of the uniform model. While a Taylor series expansion is generally used to approximate the continuous displacement, the various methods might differ for the quantities to be equated in the models. For example, Kumar and McDowell [22] selected the Cosserat description of the continuum to take into account the node rotations and the bending of the lattice elements. A second order Taylor series expansion was used for the displacements, and the coefficients of the equivalent micropolar continuum were found by comparing the expressions of the strain energy for the discrete lattice to those of the continuous medium. The suitability of the method was restricted to cell topologies with a single internal joint. Gonella and Ruzzene [16] studied the in-plane properties of hexagonal and auxetic lattices and derived the parameters of the equivalent continuum by comparing the coefficients of the in-plane wave equations of a discrete lattice to those of an equivalent continuum model. Suiker et al. [31] derived the parameters of a micropolar continuum considering the dispersion relations of harmonic waves. In both studies, a continuous model was used and provided a satisfactory approximation of the properties of the discrete lattice only for wavelengths longer than six times the dimension of the unit cell. For shorter wavelengths, local effects could not be captured. Gonella and Ruzzene [17] recently observed that the use of the Taylor series approximation at short wavelength is the main culprit for the poor accuracy of the continuous model. Since it is not possible to increase the order of the expansion, which is limited by the number of boundary conditions that can be imposed, the authors proposed a representative volume element (RVE) made of multiple unit cells. Likewise with the aim of increasing the accuracy of the continuous model in capturing the local effects of wave propagation, Lombardo and Askes [24] presented an approach based on a higher order approximation of the inertia terms only.

More recent works resorts to the application of concepts of crystals' theory. Hutchinson and Fleck [20] proposed a method based on the Bloch theorem for the statement of the equilibrium problem over an infinite lattice, and relied on the Cauchy-Born assumption to interpolate the displacement in the unit cell. The method was applied to estimate the stiffness of the Kagome and the triangular–triangular lattice. The existence of inextensional periodic collapse modes was observed for the Kagome lattice, a stretching dominated material. Elsayed and Pasini [11] extended this approach to the analysis of more generic bidimensional stretching dominated lattices. Vigliotti and Pasini [34] presented a matrix based method for the analysis of arbitrary bidimensional stretching and bending dominated lattices.

Works available in the literature are mainly focused on the evaluation of the lattice stiffness for pin-jointed open cell configurations. This paper presents a general procedure for the linear analysis of both open and closed cell three-dimensional lattices of arbitrary topology with either pin and rigid joints. The method hinges on a multiscale approach and makes no assumption on the displacements of the internal points of the unit cell; rather the change in the microscopic periodic directions is expressed as a function of the macroscopic strain filed, and the displacements of the deformed lattice are obtained by imposing equilibrium conditions. Besides stiffness, the procedure also permits the assessment of the strength of an arbitrary-cell lattice, with respect to both buckling and plastic yielding. Global buckling load and buckling modes are estimated by solving a generalized eigenvalue problem for the unit cell with prescribed periodic boundary conditions. The method is here applied to characterize the properties of 3D lattice topologies. Some topologies have been characterized here for the first time. In addition, since the method is based on the evaluation of the lattice strain energy by means of a finite element model of the unit cell, it can be extended to account for the effect of geometric and material non linearity. The method can also be applied recursively to analyse a component with multiple hierarchic levels of lattice structure.

The paper is structured as follows. First the multiscale approach is described in its general aspects in Section 2. In Section 3, the method for the lattice analysis is explained in detail. Section 4 applies the procedure for a comprehensive analysis of four lattice topologies based on the cubic unit cell. Here are given closed-form expressions of the stiffness and internal forces on lattice elements. The results of the analysis of the cubic-based topologies, and of nine lattices obtained from Archimedean solids, are finally used to develop material charts. A discussion comparing stiffness and strength properties of the lattices under investigation is given in Section 5, before the conclusion.

2. The multiscale approach

Structures built of lattice materials generally present at least two distinct length scales: the scale of the component, at the macroscopic level, and the scale of the unit cell, at the micro-level. Here, we formulate a comprehensive model consisting of two nested structural models. At the macroscopic level, the stiffness of the lattice component is determined by assuming the lattice material as a uniform continuum. At the microscopic level, we calculate the lattice stiffness and the internal forces in each lattice element, both essential for the analysis of the lattice strength.

The virtual work principle requires to equate the variation of the potential energy of the external forces to the variation of the strain energy, calculated through the constitutive relation of the material. For uniform materials, a functional relation exists between the stress and strain tensors and it reduces to the material stiffness matrix for the linear case. For lattice materials, since the relation depends on the properties of the lattice, it cannot be expressed directly as a functional dependence. Yet, a boundary value problem can be formulated to calculate the lattice strain energy and to express it as a function of the macroscopic strain. Fig. 1 summarizes the steps of the multiscale scheme, which is largely based on the work by Kouznetsova et al. [21]. At the component level (1,2), given the macroscopic displacements, $\mathbf{u}_M$, we determine the components of the Cauchy strain tensor, ϵ_M, and the deformed lattice periodic vectors, $\mathbf{a}_i$; the micro-displacements of the unit cell nodes are expressed as a function of the macroscopic strain (3); the micro-stress of the uniform solid material is obtained via the Hooke's law (4); the micro-deformation work is calculated by means of a finite element model of the unit cell (5); the macro-stress tensor is calculated as the gradient of the strain energy density with respect to the macroscopic strain (6); macro-forces are obtained applying the virtual work principle at the macroscopic level (7,8).

Two conditions are imposed to define the equilibrium problem of the unit cell: a kinematic condition to preserve the periodicity of the unit cell boundaries; a static condition to ensure the equilibrium of the confining cells. The formulation enables the analysis of a lattice with arbitrary cell topology and any number of either pin or rigid joints both on the boundary and interior of the unit cell. Once the equilibrium of the unit cell has been solved, the deformation work and the forces in the cell elements can be determined as a function of the macroscopic strain tensor.