Comput. Methods Appl. Mech. Engrg. 217–220 (2012) 1–11

Contents lists available at SciVerse ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

The DtN nonreflecting boundary condition for multiple scattering problems in the half-plane

Sebastian Acosta *, Vianey Villamizar, Bruce Malone

Department of Mathematics, Brigham Young University, Provo, UT 84602, United States

ARTICLE INFO

Article history:
Received 23 June 2011
Received in revised form 29 November 2011
Accepted 9 January 2012
Available online 16 January 2012

Keywords:
Half-plane scattering
Helmholtz equation
Nonreflecting boundary condition
Dirichlet-to-Neumann map

ABSTRACT

The multiple-Dirichlet-to-Neumann (multiple-DtN) non-reflecting boundary condition is adapted to acoustic scattering from obstacles embedded in the half-plane. The multiple-DtN map is coupled with the method of images as an alternative model for multiple acoustic scattering in the presence of acoustically soft and hard plane boundaries. As opposed to the current practice of enclosing all obstacles with a large semicircular artificial boundary that contains portion of the plane boundary, the proposed technique uses small artificial circular boundaries that only enclose the immediate vicinity of each obstacle in the half-plane. The adapted multiple-DtN condition is simultaneously imposed in each of the artificial circular boundaries. As a result the computational effort is significantly reduced. A computationally advantageous boundary value problem is numerically solved with a finite difference method supported on boundary-fitted grids. Approximate solutions to problems involving two scatterers of arbitrary geometry are presented. The proposed numerical method is validated by comparing the approximate and exact far-field patterns for the scattering from a single and from two circular obstacles in the half-plane.

1. Introduction

Wave scattering emerges in many applications such as sonar, radar, antennas, seismic exploration, crack detection, satellite imaging, and microscopy. Many of these scattering problems are more realistically modeled by taking into account the presence of infinite boundaries such as the ground surface in outdoor acoustics, radar and satellite imaging [1,2], or the surface and bottom of the ocean in marine acoustics [3–5]. The analytical solutions for acoustic scattering in the half-plane or half-space can be found only when the target conforms to simple shapes such as circles in two dimensions, or spheres in three dimensions. These solutions can be constructed using the method of images so that the problem extends to multiple scattering in the full-space or full-plane. Then, modal expansions in separable coordinates systems or explicit evaluations of integral representations are available. For details, see [1,6], [7, Appendix A], or the book by Martin [8] with its extensive reference list.

For arbitrarily shaped obstacles, a closed form solution is not generally found, although some useful asymptotic approximations have been constructed using integral representations in the low frequency regime [8–10]. Hence, the scattering from an obstacle of complex geometry embedded in the half-space generally requires the application of numerical methods. One class of such methods is based on boundary integral formulations. We direct the reader to [8, Chapters 5 and 6] for an excellent overview and reference list of integral equation methods employed in multiple scattering theory. Some of the well-known advantages of these methods are the reduction in dimensionality (from volume to surface) of the domain of discretization, the automatic satisfaction of the radiation condition at infinity and of the boundary condition on the plane boundary for semi-infinite media. This is accomplished by using the correct Green's function as the integral kernel. However, these methods may become quite costly since they lead to dense matrices and the reduction of dimensionality is lost if the media contains inhomogeneities. Another important class of numerical methods for scattering problems are based on finite elements and finite differences. These volume discretization techniques, as opposed to boundary integral methods, are naturally suited for treating localized heterogeneities, nonlinearities and sources, as well as the presence of impenetrable obstacles. Their major drawback, however, is the handling of the unboundedness of the domain.

A great deal of research has been performed to formulate appropriate absorbing boundary conditions in order to truncate the physical domain. These boundary conditions should allow the outgoing waves to leave the truncated domain without nonphysical reflections. Some of these conditions include local absorbing boundary conditions [11–14], perfectly matched layers [15–18] and an exact nonreflecting boundary condition known as

* Corresponding author. Tel.: +1 801 361 9833.
E-mail addresses: sebastian@math.byu.edu (S. Acosta), vianey@math.byu.edu (V. Villamizar).

0045-7825/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.cma.2012.01.005

Dirichlet-to-Neumann (DtN) map [19–21]. The potential advantages and drawbacks of these conditions have been extensively studied by Givoli [22–24] and Tsynkov [25]. Some are easier to implement, others are more robust, while others may be computationally more efficient. However, a general agreement in favor of one of these conditions over the others has not been reached.

For the full-plane, the most common practice has been to enclose all scatterers with a single artificial circular or elliptical boundary and apply the absorbing boundary condition on it. For half-plane problems, multiple scattering interactions between the plane boundary and the scatterers take place. For that reason, the artificial boundary typically consists of a semi-circular or elliptical artificial boundary enclosing all scatterers and a portion of the plane boundary. This was the approach followed by Lee and Kallivokas [7], and Givoli and Vigdergauz [26] where excellent numerical results for the scattering from two-dimensional obstacles were obtained. For instance, an effort was made in [7] to extend the applicability of elliptically shaped absorbing boundaries to half-plane problems. Its use rendered significant computational savings for elongated obstacles near the plane boundary in comparison with the use of semi-circular boundaries. However, if the obstacle is relatively far from the plane boundary or if the scatterer consists of several disjoint obstacles, the use of a single artificial boundary (circular or elliptical) to enclose all the obstacles and a portion of the plane boundary will inevitably lead to a large computational domain.

An improvement can be made in terms of reducing the size of the computational domain. In fact, Grote and Kirsch [27] introduced the multiple-DtN map as a nonreflecting boundary condition for the scattering from several obstacles embedded in the full-space or full-plane. Later, Acosta and Villamizar [28] combined it with curvilinear coordinates and applied it to obstacles of arbitrary shape. Its definition requires the introduction of separate artificial boundaries, each one enclosing a different obstacle, reducing the computational region to a set of small sub-domains. Then, the multiple-DtN condition is simultaneously imposed on each artificial boundary. The net effect of this condition is that propagating waves are allowed to leave the computational sub-domains without spurious reflections and simultaneously account for the wave interactions between the different sub-domains. As mentioned in [27], neither local absorbing boundary conditions nor perfectly matched layers in their current form successfully deal with such multiple scattering interactions.

The purpose of the present work is to adapt the multiple-DtN technique, described above for scattering in the full-plane, to scattering problems in the half-plane. The main idea can be conveniently illustrated by Fig. 1. Here, two domains corresponding to equivalent problems are depicted. In Fig. 1(a), the semi-infinite domain Ω of the *original half-plane problem*, internally bounded by $\mathcal{C}$ and lying above the plane boundary Γ, is shown. On the other hand in Fig. 1(b), the small annular domain Ω_{int} of the final problem, bounded internally by $\mathcal{C}$ and externally by the artificial circular boundary $\mathcal{B}$, is also shown. The proposed method based on an adaptation of the multiple-DtN technique requires: First, the construction of a new multiple-DtN condition for the half-plane which handles the interactions between the plane boundary and the scatterer. Secondly, the approximation of the solution of the unbounded original problem by numerically solving the equivalent final problem in the bounded domain Ω_{int} with the novel multiple-DtN condition imposed on the artificial circular boundary $\mathcal{B}$. This final problem will be called *half-plane multiple-DtN scattering problem* in this work.

The derivation of the adapted condition involves several steps. To start, consider the physical problem of scattering from a single obstacle in the half-plane with an acoustically soft or hard boundary condition on the plane boundary Γ. A natural approach consists of using the *method of images* to extend the half-plane problem to a multiple scattering problem containing two scatterers in the full-plane. This problem is in turn reduced to an equivalent two-obstacle bounded BVP in the full-plane by using the multiple-DtN map [27], as explained above. The intrinsic symmetry of the method of images is exploited for a further reduction of the two-obstacle bounded problem to a single-obstacle BVP defined in the small bounded domain Ω_{int} of the half-plane, as shown in Fig. 1(b). The symmetry also leads to a natural derivation of the new multiple-DtN condition for the half-plane from the corresponding multiple-DtN condition in the full-plane. The derivation of this condition and its extension to a finite number of obstacles in the half-plane constitute one of the main contribution of this work.

The final BVP with the novel multiple-DtN boundary condition, defined in the small domain Ω_{int} of the half-plane, can be accurate and efficiently solved by using appropriate numerical volume methods such as finite elements or finite differences. The radius of the exterior artificial circular boundary $\mathcal{B}$ of the small domain Ω_{int} is independent of the distance between the obstacle and the plane boundary. Since the scattering interactions between the plane boundary and the obstacle are handled by the new multiple-DtN condition, there is no need to retain the interacting portion of the plane boundary within the truncated domain. As a result, the computational region is significantly reduced in comparison with the use of a large semi-circular artificial boundary enclosing not only the obstacle, but also a portion of the plane boundary. This is significant in the simulation of sonar and radar problems. For instance, the distance between a submarine and the bottom of the ocean may be several times larger than the characteristic length of the submarine.

The multiple-DtN boundary condition is independent of the numerical discretization employed in the truncated domain. It has been successfully implemented using finite element (FEM)

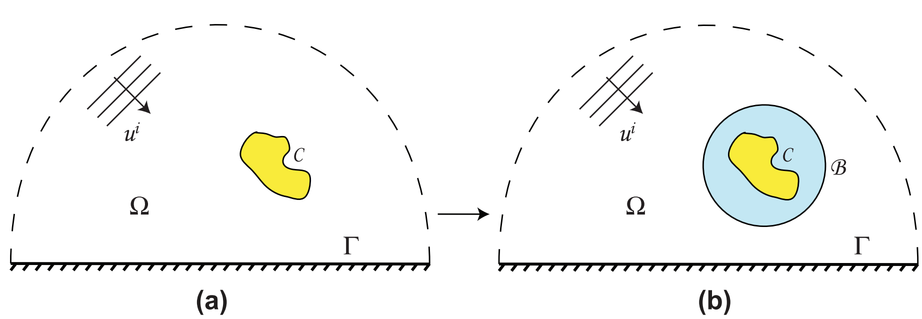

Fig. 1. (a) Original physical scattering problem, (b) Truncated domain where the new multiple-DtN condition will be imposed and the numerical approximations to the original problem will be calculated.