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Uncertainty based robust optimization method for drag minimization problems in aerodynamics ☆

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ABSTRACT

A new robust optimization method is introduced to extend single point design to more realistic problems in aerodynamics taking into account uncertainties. It is well known that single point design techniques produce solutions that perform well for the selected design point but have poor off-design performance. Following ideas of Taguchi's robust control theory, a design with uncertainties is replaced by an optimization problem with two objectives which are mean performance and variance. Here, this two-objective optimization problem is solved by Pareto and Nash game strategies combined with the adjoint method, in the sense that solutions are less sensitive to uncertainties of input parameters. A constrained Nash strategy is implemented for performing multi-criteria optimization problems with constraints. Starting from a statistical definition of stability, the method simultaneously captures, Pareto and Nash equilibrium solutions ensuring performance and stability.

1. Introduction

In many industrial applications, some values of input parameters cannot be provided accurately. For example, uncertainties could characterize some geometry entities (lengths, relative positions, angles…) that concerns the problem studied. Quite often the operating flight conditions are not fixed, but *vary* in the presence of fluctuations. For these reasons, many input parameters can be estimated by mean and variance values [1].

In aircraft design, an important task in the shape optimization of airfoils is the sensitivity of the final optimal design to small manufacturing errors or fluctuations during the operating conditions. Practical experience with airfoil optimization techniques has revealed unexpected difficulties. Traditionally the performance of an airfoil is optimized at a fixed operating condition. Experience has indicated that the optimization may result in dramatically inferior performance when the actual conditions are differ from the fixed design condition [2–4]. Tightening the tolerances in the manufacturing process may prove prohibitively expensive or practically impossible to achieve. Moreover, *certain* variability in operating conditions (e.g. flight speed) cannot be avoided. Developing optimization methods which result in more *robust* designs sounds more challenging [5–7].

Several independent approaches (multi-point, bounds-based, minimax, fuzzy and probabilistic methods, Taguchi methods, etc.) can be considered to achieve *robustness* and a detailed review of such approaches is given in [8–10].

A straightforward approach taking into account uncertainties is to generalize the objective function as a linear combination of uncertain conditions, i.e. multi-point optimization. Practical problems arise with the selection of uncertain conditions and with the specification of the weights. There is no clear theoretical principles to guide the selection, which is in fact largely left up to the designer's discretization (see [11,12]). With the multi-point formulation, an improved C_d can be realized over a wider range of Mach numbers M_∞. However, this formulation is still unable to provide a truly global solution by avoiding localized optimization. In fact multiple *bumps* appear on the airfoil, one associated with each flight condition $M_{\infty,i}$. In the transonic regime, each bump occurs at the shock foot location for each of sampled Mach numbers.

Minimax method [13] is another approach taking into account uncertainties by using an adaptive weight, then to minimize the maximum value of $\rho(M)C_d(M)$. The adaptive weight $\rho(M)$ and $C_d(M)$ are the functions of uncertainty, i.e. Mach number. The algorithm terminates when it is impossible to achieve a consistent drag reduction at all design Mach numbers. For a particular sequence of weights, it can be shown to revert to multi-point optimization algorithms. In a practical numerical implementation, the method also requires an priori selection of design conditions.

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According to the Von Neumann–Morgenstern statistical decision theory, the best course of action in the presence of uncertainty is to select the design which leads to the lowest expected drag. This is commonly known as the Maximum Expected Value criterion (MEV). The risk, associated with a particular design, is identified as the expected value of the perceived loss associated with the design [14,15]. The best design or decision, which minimizes the overall risk, is referred to as the *Bayes' decision*. The practical problem is that integration is required in each optimization step in order to compute the Bayes' risk and Bayes' decision. Since the objective function C_d is computationally expensive to evaluate, this approach, although theoretically sound, becomes prohibitively expensive.

Taguchi, a Japanese engineer, had a significant contribution on quality control and experimental design in the 1980s and 1990s. He suggested that *quality* should be achieved, not as a product being inside or outside specifications, but from the variation from the target. Taguchi recommended a two-step design process to *reduce* variation in a product by reducing the sensitivity of the design of the product to sources of variation rather than by controlling their sources.

In this paper, the Taguchi robust optimization concept [16] is introduced to solve single point design problems with uncertainties. The latter techniques provide solutions that perform well at selected design point but have poor off-design performance. In this study, we show how it is possible to set an airfoil optimization problem at transonic flight conditions to capture numerically robust solutions, in the sense that the solutions are as insensitive as possible to small changes of input parameters. Starting from a statistical definition of stability, the method finds simultaneously admissible solutions for performance and stability. So the Taguchi robust design method aims to look for a solution with better mean performance and less variance of performance within a certain interval where input parameters can vary. Pediroda et al. [17] investigated robust aerodynamic design problem by combining a Multi Objective Genetic Algorithm with a Taguchi concept, but this evolutionary procedure is time consuming. Here, the multi objective deterministic optimization method based on game strategies and adjoint equations [18] is used to solve robust design problem more efficiently. In order to treat robust design with constraints, a constrained Nash equilibrium computation method is implemented, in which the partial gradient, instead of design variables, is used as elite information exchange in the Nash algorithm.

The proposed Taguchi methodology is tested and evaluated for the drag minimization of an airfoil operating in Euler flows with a variable Mach number at transonic regime. Then the mean drag and its stability are introduced as objective functions. We selected this 2-D model problem in aerodynamics since it is well known that the answer of this problem with fluctuations at operated conditions is highly non-linear due to the presence of shock waves.

It will be concluded from the obtained results that a robust optimization technique is needed when some of parameters, like operating flight conditions, are fluctuating randomly within specified design intervals and above all when uncertainties have a strong nonlinear effect on the behavior of objective functions.

2. Robust control and design optimization

2.1. Robust control

Many products are now routinely designed with the aid of computer models. Given inputs designable engineering parameters and parameters representing manufacturing process conditions, the model generates the product's quality characteristics. The quality improvement problem requires *choosing* designable engineering parameters such that quality characteristics are uniformly stabilized in the presence of variability in processing conditions.

Let $f(x,b):\Re^n\otimes\Re^m\rightarrow\Re$ be a real-valued function. Where $x\in X\subset\Re^n$ represents decision variables, inputs (designs) controlled by the engineer, and $b\in B\subset\Re^m$ represents uncertainty, inputs not controlled by the engineer. $f(x,b)$ quantifies the loss that accrues from design x when conditions b obtain. By *robust control* we mean the problem of finding $x^*\in\Re^n$ such that $f(x^*,b)$ is as uniform smaller as possible when b varies in B [16]. By this definition we mean more than just the local behavior that any continuous f will necessarily exhibit. Robust control is concerned with semi-local behavior and our goal is to avoid simply choosing x^* as a local minimum of f if that *minimum* lies at the bottom of a very steep and narrow basin.

Minimax principle: Robust optimization is inherently multi-objective. One way of achieving our objectives is to replace $f(x,b)$ with its worst-case on B. A very conservative possibility is then to measure the performance of $f(x,b)$ on B by its supremum, resulting in the robust control problem

$$\min_{x\in X}\max_{b\in B} f(x,b). \tag{1}$$

This formulation protects against worst-case *scenarios* and is analogous to the minimax principle in statistical decision theory and elsewhere. Minimax optimization problems have been the subject of considerable study, e.g. by Dem'yanov and Malozemov [13], but they are not smooth and are likely too conservative for the present application.

Bayes principle: An alternative is to measure the performance of f on B by some mean value, so that our objective becomes that of minimizing some kind of decreasing average of f. This can be accomplished by defining a weighting function (presumably a probability density function named PDF) $p:B\rightarrow\Re$, resulting in the robust optimization problem

$$\min_{x\in X}\int_B f(x,b)p(b)db. \tag{2}$$

This formulation addresses typical (as defined by p) *scenarios* and is analogous to the Bayes principle in statistical decision theory.

Example: Aircraft Shape Design. Suppose that f is the drag coefficient, and x the shape of an airplane. Then b might specify:

1. Manufacturing errors ε that perturb the design. The desired airplane shape is x, but the manufactured aircraft shape is $[x-\varepsilon,x+\varepsilon]$. The problem is to find a design that will minimize the drag of the manufactured aircraft. This problem has been studied by Welch and Sacks [19].
2. Mach numbers $M\in[M_b-\varepsilon,M_b+\varepsilon]$, angle of attack $\alpha\in[\alpha_b-\epsilon,\alpha_b+\epsilon]$. The problem is to design an aerodynamic shape that performs well over a range of different Mach numbers and angles of attack. This problem has been considered by several researchers at NASA; the approach, has been pursued by Huyse and Lewis [2], Tang [3] and Huyse [6,8,14,15], is closely related to ours.

Following Bayes principle, let $x\in X\subset\Re^n$ denote the design.

1. Let $b=\varepsilon\in B\subset\Re^m$ denote the manufacturing error and let $f(x,b)=f(x-\varepsilon)$ denote the drag coefficient of the manufactured wing or aircraft. Then, minimize

$$\phi(x)=\int_{\Re^m} f(x-\varepsilon)p(\varepsilon)d\varepsilon=[f\otimes p](x). \tag{3}$$

In this case, p might be chosen to approximate the probability distribution of random manufacturing errors that perturb the design.