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Disjoint Hypercyclic Powers of Weighted Pseudo-Shifts

Ya Wang[1] · **Ze-Hua Zhou**[1]

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Abstract In this paper, we characterize disjoint hypercyclic powers of weighted pseudo-shifts on an arbitrary F-sequence space. As a special case, we deduce the equivalent conditions for the disjoint hypercyclicity of finitely many different powers of weighted shifts on $\ell^2(\mathbb{Z}, \mathcal{K})$ with weight sequence $\{A_n\}_{n=-\infty}^{\infty}$ of positive invertible diagonal operators on $\mathcal{K}$.

Keywords Disjoint hypercyclic · Weighted pseudo-shifts · Operator weighted shifts

Mathematics Subject Classification 47A16 · 47B38 · 46E15

1 Introduction

Let $\mathbb{N}$ denote the set of nonnegative integers and $\mathbb{Z}$ denote the set of all integers. Let X be a separable infinite dimensional F-space over $\mathbb{K} = \mathbb{R}$ or $\mathbb{C}$, and the algebra of all continuous linear operators on X will be denoted by $L(X)$. Let $T \in L(X)$ and T^n denote the n-th iterate of T. The operator T is said to be *hypercyclic* if there is some

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✉ Ze-Hua Zhou
zehuazhoumath@aliyun.com; zhzhou@tju.edu.cn

Ya Wang
wangyasjxsy0802@163.com

1 School of Mathematics, Tianjin University, Tianjin 300354, People's Republic of China

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vector $x \in X$ such that the orbit $\mathrm{Orb}(T, x) = \{T^n x : n \in \mathbb{N}\}$ is dense in X. Such a vector x is said to be hypercyclic for T.

The first example of a hypercyclic operator on a Banach space was offered in 1969 by Rolewicz [16], who showed that if B is the unilateral backward shift on $\ell^2(\mathbb{N})$, then λB is hypercyclic if and only if $|\lambda| > 1$. After that, Salas [19] completely characterized the hypercyclic unilateral weighted backward shifts on $\ell^p(\mathbb{N})$ with $1 \leq p < \infty$ and the bilateral weighted shifts on $\ell^p(\mathbb{Z})$ with $1 \leq p < \infty$ in terms of their weight sequences. León-Saavedra and Montes-Rodríguez [14] later used Salas' weight characterization to show that each type of weighted shifts is hypercyclic precisely when it satisfies the so-called Hypercyclicity Criterion. This criterion plays a key role in the theory of hypercyclic operators. It was obtained independently by Kitai [12] and by Gethner and Shapiro [7], and it provides a sufficient condition for a general operator to be hypercyclic. Using the Hypercyclicity Criterion, Grosse-Erdmann [9] and Hazarika and Arora [11] extended Salas' results independently. In [9] Grosse-Erdmann obtained a characterization for hypercyclic weighted pseudo-shifts on an arbitrary F-sequence space. In [11], Hazarika and Arora considered the hypercyclic bilateral operator weighted shifts on $\ell^2(\mathbb{Z}, \mathcal{K})$ with weight sequence $\{A_n\}_{n=-\infty}^{\infty}$ of positive invertible diagonal operators on $\mathcal{K}$. For more examples and background about hypercyclic operators, we refer the readers to the excellent books by Bayart and Matheron [2], and by Grosse-Erdmann and Peris Manguillot [8].

In 2007, Bès and Peris [5] and Bernal-González [1] introduced the disjointness in hypercyclicity independently. For any integer $N \geq 2$, we say that hypercyclic operators $T_1, \ldots, T_N$ in $L(X)$ are *disjoint hypercyclic* or *d-hypercyclic* if their direct sum $T_1 \oplus \cdots \oplus T_N$ has a hypercyclic vector of the form $(x, \ldots, x)$ in X^N, x is called a *d-hypercyclic* vector for $T_1, T_2, \ldots, T_N$. If the set of d-hypercyclic vectors is dense in X, we say $T_1, T_2, \ldots, T_N$ are *densely d-hypercyclic*. In 2010, Shkarin [18] provided a brief proof for the existence of disjoint hypercyclic operators. Salas [17] showed that if the separable Banach space X have separable dual space, then we can find disjoint hypercyclic operators on X such that their adjoint operators are also disjoint hypercyclic. In [5], Bès and Peris extended the Hypercyclicity Criterion to the disjoint setting, called the *d-Hypercyclicity Criterion*, which provides a sufficient condition for *d-hypercyclicity*. In the same paper, they also established the following characterization for a finite family of different powers of weighted shifts to be d-hypercyclic.

Theorem 1.1 [5, *Theorem* 4.7] *Let $X = c_0(\mathbb{Z})$ or $\ell^p(\mathbb{Z})$ $(1 \leq p < \infty)$. For $l = 1, \ldots, N$, let $a_l = (a_{l,j})_{j\in\mathbb{Z}}$ be a bounded bilateral sequence of nonzero scalars, and let B_{a_l} be the associated backward shift on X given by $B_{a_l} e_k = a_{l,k} e_{k-1} (k \in \mathbb{Z})$. For any integers $1 \leq r_1 < r_2 < \cdots\cdots < r_N$, the following are equivalent:*

(a) *$B_{a_1}^{r_1}, B_{a_2}^{r_2}, \ldots, B_{a_N}^{r_N}$ have a dense set of d-hypercyclic vectors.*

(b) *For each $\epsilon > 0$ and $q \in \mathbb{N}$, there exists $m \in \mathbb{N}$ so that for $|j| \leq q$ we have :*

$$\text{If } 1 \leq l \leq N, \quad \begin{cases} |\prod_{i=j+1}^{j+r_l m} a_{l,i}| > \frac{1}{\epsilon}, \\ |\prod_{i=j-r_l m+1}^{j} a_{l,i}| < \epsilon. \end{cases}$$

$$\text{If } 1 \leq s < l \leq N, \quad \begin{cases} |\prod_{i=j+1}^{j+r_l m} a_{l,i}| > \frac{1}{\epsilon} |\prod_{i=j+(r_l - r_s)m+1}^{j+r_l m} a_{s,i}|, \\ |\prod_{i=j-(r_l-r_s)m+1}^{j+r_s m} a_{l,i}| < \epsilon |\prod_{i=j+1}^{j+r_s m} a_{s,i}|. \end{cases}$$