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A hierarchical approach to adaptive local refinement in isogeometric analysis

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ABSTRACT

Adaptive local refinement is one of the key issues in Isogeometric Analysis. In this article we present an adaptive local refinement technique for isogeometric analysis based on extensions of hierarchical B-splines. We investigate the theoretical properties of the spline space to ensure fundamental properties like linear independence and partition of unity. Furthermore, we use concepts well-established in finite element analysis to fully integrate hierarchical spline spaces into the isogeometric setting. This also allows us to access a posteriori error estimation techniques. Numerical results for several different examples are given and they turn out to be very promising.

1. Introduction

Adaptive splines which provide local refinement has been studied as an effective tool for surface modeling. Adaptive refinement of spline basis functions has also recently become an active research topic within the framework of isogeometric analysis [1,2], which combines numerical simulation with computer aided design (CAD) geometry by using exact CAD representations like non-uniform rational B-splines (NURBS) into the analysis model. Isogeometric analysis has been applied to a wide range of problems from fluid–structure interaction [3], shape optimization [4], shell analysis [5] to electromagnetics [6], just to name a few. Nevertheless, local refinement is still a key issue due to the tensor-product structure which makes it difficult to obtain finer grids without propagation of the refinement.

Particular attention has been devoted to the application of the promising concept of T-splines [7,8] into the isogeometric context [9,10]. T-splines allow to break the rigidity of the rectangular topology which characterizes the standard NURBS model, currently used in commercial CAD systems, by considering a mesh – generally indicated as T-mesh – in the parameter domain with axis-aligned edges, where T-junctions (similar to hanging nodes in standard finite elements) are permitted. However, the need to overcome certain limitations of T-splines, in particular with respect to the linear dependence of T-spline blending functions corresponding to particular T-meshes [11] and to the locality of the refinement [9], requires further investigations for the identification of geometric representations suitable for analysis.

For this reason, the most recent developments in the field of T-splines include the introduction of "analysis-suitable" T-spline spaces [12]. This restricted set of T-splines allows to guarantee linear independence and restricts the number of additional control points generated by the refinement algorithm to a minimum. These kind of T-spline spaces, however, need a more complicated refinement algorithm, and they still require additional knot insertions beyond the ones specified by the application (by the user and/or by an error estimator).

Other authors consider hierarchical spline spaces over T-meshes with reduced regularity, such as bicubic C^1 splines [13,14]. Once again, these spaces can be considered as special T-splines [11]. These splines are closely related to classical Hermite elements (e.g. bicubic rectangles) for FEM with hanging nodes, and recently, they were used for isogeometric simulation [15]. The required local behavior of the refinement algorithm and linear independence can be guaranteed easily. The price to pay for this, however, is the reduced regularity of the basis, which increases the number of degrees of freedom needed to obtain the same accuracy.

In order to allow local editing of tensor-product spline surfaces at different levels of detail, Forsey and Bartels [16] introduced hierarchical B-splines as an accumulation of tensor-product splines with nested knot vectors. More precisely, they modified existing surfaces by locally adding patches representing finer details. Later, Kraft [17] identified a basis for the spaces spanned by these splines and considered their stability. So far, however, these hierarchical splines have found only few applications in geometric design and

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no applications in isogeometric analysis. The hierarchical model allows complete local control of the refinement by using a spline hierarchy whose levels identify subsequent levels of refinement for the underlying geometric representation.

Working with hierarchies of finite-dimensional subspaces offers also a number of advantages from the numerical point of view. For one, there is a straightforward connection to state-of-the-art iterative solvers and preconditioning techniques for large-scale linear systems [18], and furthermore, adaptive refinement and numerical linear algebra may be tightly linked at the algorithmic level [19].

Besides preserving the exact geometry, one of the attractive features of isogeometric analysis is that it offers basis functions with higher smoothness than in standard finite elements [20]. This extra smoothness leads in many cases to better convergence properties [21] and is also valuable when dealing with partial differential equations that demand for H^2-regularity, such as plate and certain shell problems. Accordingly, for a general local refinement approach it is vital that it can handle and inherit any given degree of continuity. Hierarchical spline spaces are able to meet this requirement. On the other hand, the usage of multiple knots that lead to a decrease of smoothness is also possible.

After this prelude on the motivation for our work, we present in this article the idea of hierarchical B-splines for local adaptive refinement in isogeometric analysis. We rely on a sound theoretical foundation supporting the spline space as well as established finite element concepts and a posteriori error estimation to elaborate this approach. Key properties such as linear independence of the corresponding basis, locality of the refinement process, and arbitrary smoothness can be accomplished in this way, and, moreover, the first computational examples turn out to be promising.

Before starting with the comprehensive exposition we give a short example to illustrate the underlying idea. In Fig. 1, a one-dimensional refinement step is shown. At the top, quadratic B-splines are plotted for a given equidistant knot vector. The plot in the middle displays the B-splines after each knot span has been subdivided by knot insertion. The aim now is to locally refine the right half of the original B-splines whereas the left half remains coarse. Obviously, this could also be realized by means of knot insertion, but in multiple dimensions this would result in a propagation of the refinement due to the tensor-product structure. Instead, we replace the coarse basis function which are located at the right (marked by a dashed line) by fine basis function (marked by a solid line), and in this way we create the basis shown at the bottom. The appropriate combination of basis function at different refinement levels is the very simple and powerful idea of hierarchical refinement. The generalization of this process to more complex settings and its properties will be discussed in the following.

The outline of this article is as follows. In Section 2, the hierarchical B-splines are introduced and discussed from a general geometric point of view. Especially it will be shown that favorable properties can be proved and hold for this basis by construction. In Section 3 the hierarchical concept is combined with the framework of isogeometric analysis. To ensure compatibility with finite element routines, the equivalent of an element is introduced and active elements and basis functions at the different levels are defined. An adaptive isogeometric method is then outlined, equipped with standard techniques for error estimation and an extension of the marking algorithm for selecting the regions to be refined. The refinement behavior is furthermore investigated, and it is demonstrated that no undesirable insertion of extra grid points may occur. In Section 4, finally, the hierarchical refinement is applied to several examples while conclusions are drawn in Section 5.

2. Hierarchical splines

The only requirement of the philosophy which characterizes the hierarchical approach is a *refinable* nature of the underlying basis functions defined on nested approximation spaces. Local control of the refinement is achieved through an adaptive procedure that is exclusively based on basis refinement. In this work we consider hierarchical B-spline spaces, but others family of basis functions which exhibit analogous properties and similarly allow adaptive refinement may also be used to define spline hierarchies suitable for analysis.

After introducing some basic notions, in this section we show how to construct a piecewise polynomial non-negative basis composed by locally supported basis functions which can also be modified to form a partition of unity. Moreover, we outline that the hierarchical construction of the spline basis directly implies a nested nature of the corresponding space hierarchy.

2.1. Tensor product B-spline spaces

A bivariate tensor-product B-spline space $\mathcal{B}$ is defined by specifying the polynomial degree (p,q) and the horizontal and vertical knots vectors

$$U = \{u_0 \leqslant u_1 \leqslant \cdots \leqslant u_{n+p+1}\},\ V = \{v_0 \leqslant v_1 \leqslant \cdots \leqslant v_{m+q+1}\},$$

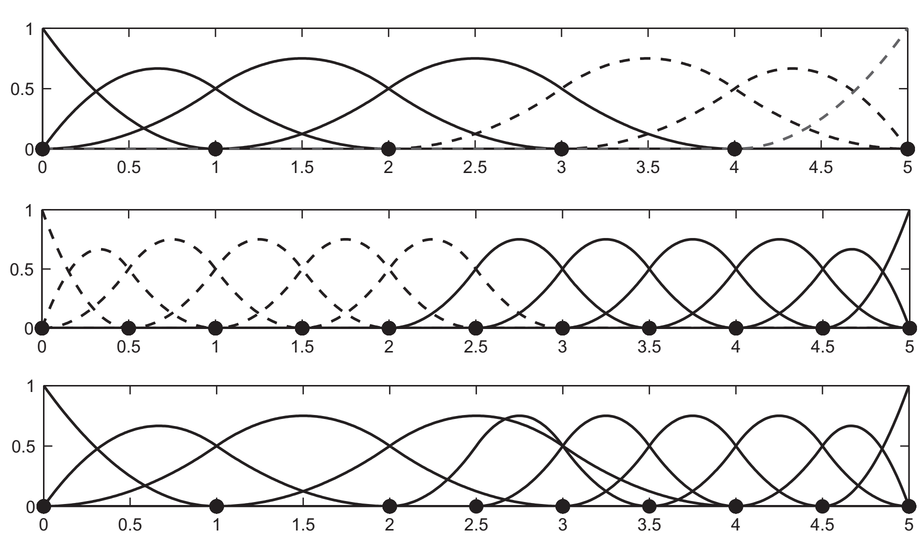

Fig. 1. Initial basis (above), refined basis (middle), suitable combination of both (below); selected basis function are marked by a solid line.