Journal of Colloid and Interface Science 503 (2017) 103–114

Contents lists available at ScienceDirect

Journal of Colloid and Interface Science

journal homepage: www.elsevier.com/locate/jcis

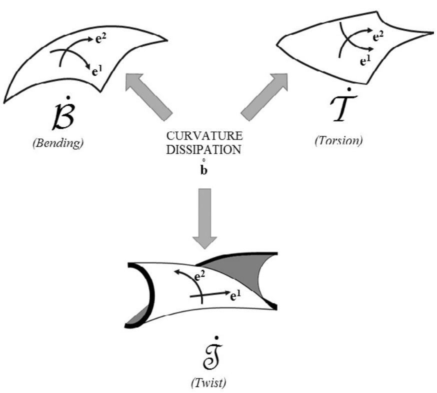
Regular Article

Generalized Boussinesq-Scriven surface fluid model with curvature dissipation for liquid surfaces and membranes

Oscar F. Aguilar Gutierrez [a], Edtson E. Herrera Valencia [b], Alejandro D. Rey [a,*]

[a] Department of Chemical Engineering, McGill University, 3610 University Street, Montreal, Quebec H3A 0C5, Canada
[b] Departamento de Ingeniería Química, FES Zaragoza UNAM, Av. Guelatao No. 66 Col. Ejército de Oriente, Iztapalapa CP 09230, Mexico

GRAPHICAL ABSTRACT



ARTICLE INFO

Article history:
Received 28 July 2016
Revised 28 April 2017
Accepted 4 May 2017
Available online 8 May 2017

Keywords:
Curvature dissipation
Interfacial rheology
Rheological constitutive equation
Bending torsion twist modes

ABSTRACT

Curvature dissipation is relevant in synthetic and biological processes, from fluctuations in semi-flexible polymer solutions, to buckling of liquid columns, to membrane cell wall functioning. We present a micromechanical model of curvature dissipation relevant to fluid membranes and liquid surfaces based on a parallel surface parameterization and a stress constitutive equation appropriate for anisotropic fluids and fluid membranes. The derived model, aimed at high curvature and high rate of change of curvature in liquid surfaces and membranes, introduces additional viscous modes not included in the widely used 2D Boussinesq-Scriven rheological constitutive equation for surface fluids. The kinematic tensors that emerge from the parallel surface parameterization are the interfacial rate of deformation and the surface co-rotational Zaremba-Jaumann derivative of the curvature, which are used to classify all possible dissipative planar and non-planar modes. The curvature dissipation function that accounts for bending, torsion and twist rates is derived and analyzed under several constraints, including the important inextensional bending mode. A representative application of the curvature dissipation model to the periodic oscillation in nano-wrinkled outer hair cells show how and why curvature dissipation decreases with frequency, and why the 100 kHz frequency range is selected. These results contribute to characterize curvature dissipation in membranes and liquid surfaces.

* Corresponding author.
E-mail address: alejandro.rey@mcgill.ca (A.D. Rey).

http://dx.doi.org/10.1016/j.jcis.2017.05.012
0021-9797/

Nomenclature

Abbreviations

{1D, 2D, 3D} one, two and three dimension
OHC outer hair cell
TIF transversely isotropic fluid
nm nanometers

Subscripts

S surface
BS Boussinesq-Scriven

Notations

A area of a solid shell [m^2]
b characteristic frequency [rad/s]
$\{c_1, c_2\}$ principal curvatures [1/m]
$\{c_m, e_m\}$ eigenvalues and eigenvectors of b [1/m, 1]
$c_s = \partial M/\partial s$ arclength partial derivative [Pa m^2/ m]
D deviatoric curvature [1/m]
De Deborah number [1]
D_i diameter [m]
$F_c(t)$ scalar compression force [N/m]
$\{F_0, F_1\}$ Euler buckling threshold and amplitude oscillatory driving force [N/m]
H_I hypergeometric integral formula [1]
h_1 interface/membrane thickness [m]
h vertical displacement [m]
{h(0,t), h(L,t)} vertical displacement at x = 0 and x = L [m]
Δh fractional height change [1]
$\{\Delta h_{min}, \Delta h_{max}\}$ fractional height change at t_{min} and t_{max} [1]
H mean surface curvature [1/m]
K Gaussian curvature [1/m^2]
k_c membrane bending rigidity elastic moduli [J]
M scalar viscoelastic moment [Pa m^2]
Y undetermined constant of the amplitude [m]
r_m principal curvature radii [m]
R(t) radius of curvature [m]
R(t = 0) initial radius of curvature [m]
$R = \Delta c/\Delta_{BS}$ rate of change of curvature dissipation/deformation rate dissipation [1]
$R_{uniaxial}$ ratio of curvature uniaxial [1]
$R_{biaxial}$ ratio of curvature biaxial [1]
$R_{triaxial}$ ratio of curvature triaxial [1]
r_m principal radii of curvature [m]
r(t) radius [m]
s arc-length [m]
t time [s]
t_o initial time [s]

Greek letters

$\{\alpha_{00}, \alpha_{01}, \alpha_1, \alpha_4, \alpha_{56}\}$ viscosities [Pa s]
Δ_{BS} Boussinesq-Scriven dissipation function [Pa m /s]
Δ_{GBS} generalized Boussinesq-Scriven dissipation function [Pa m/s]
Δ_C curvature dissipation function [Pa m/s]
$\langle\overline{\Delta}^*\rangle$ dimensionless space-averaged curvature dissipation per cycle [1]
η solvent viscosity per unit length [Pa s m^{-1}]
ηD_i^3 bending viscosity [Pa m^2s]
η_{in} internal friction force [J s]
η_m bulk viscosity [Pa s]
$\{\eta^{\mathcal{B}}, \eta^{\mathcal{T}}, \eta^{T}\}$ bending, torsion and twist viscosities [J s]
η^b bending viscosity [J s]
η^{tt} Torsion-twist viscosity [J s]
$\Gamma(x)$ Gamma function [1]
κ^s surface dilatational viscosity [Pa m s]
μ^s surface shear viscosity [Pa m s]
$\{\theta^{As}, \theta^{As} \pm \pi/2\}$ principal directions [1]
$\{\theta^{b^\circ}, \theta^{b^\circ} \pm \pi/2\}$ principal directions [1]
λ distance from parent surface [m]
$\{\lambda_2, \lambda_1\}$ upper and lower limits of cross-thickness integral [m]
τ_n polymer chain relaxation time [s]
τ period frequency [s]
τ' integration variable [1]
$\phi(s,t)$ normal angle [rad]
Ω_n angular velocity [rad/s]
ω angular frequency [rad/s]

Vector, dyadic and tensors

A rate of strain tensor [1/s]
$\mathbf{A_S}$ symmetric surface rate of strain tensor [1/s]
b curvature tensor [1/m]
C symmetric tensor C [1]
D**q** deviatoric curvature tensor [1/m]
$\{\mathbf{e_1}, \mathbf{e_2}\}$ main curvature frame [1/m]
$\mathbf{I_S}$ surface unit normal [1]
$\mathbf{I_S I_S}$ dyadic product of the surface unit normal [1]
I unit dyadic tensor [1]
H**Is** trace curvature tensor [1/m]
$\{\mathbf{I_s}, \mathbf{q}, \varepsilon_s, \mathbf{q_1}\}$ four independent basis surface tensor [1]
k surface unit normal vector [1]
M viscous torque [J]
Ms viscous interface moment tensor [Pa m^2]
$\{\mathbf{M}_s^{\mathcal{B}}, \mathbf{M}_s^{\mathcal{T}}, \mathbf{M}_s^{T}\}$ symmetric viscous bending, torsion and twist moment tensors [J/m]
n director vector [1]
nn dyadic product of the director vector [1]
P second order tensor [1]
q surface basis tensor [1]
qq dyadic product of q tensor [1]
$\mathbf{q_1 q_1}$ dyadic product of q_1 surface [1]
$\mathbf{q_n}$ wave vector [1/m]
r(***s***) space curve [m]
$\mathbf{r}_\perp$ transverse displacement [m]
t unit tangent vector [1]
tt dyadic product of the unit tangent vector [1]
tk dyadic product between of the unit tangent and normal vectors [1]
T symmetric viscous stress tensor [Pa]
$\mathbf{T^T}$ transpose of a symmetric viscous stress tensor [Pa]
$\mathbf{T}_S$ tangential viscous stress tensor [Pa m]
U tangential component of the velocity field vector v [m/s]
v interface velocity vector [m/s]
$V^n\mathbf{k}$ normal component of the vector v [m/s]
Ws parent Surface vorticity tensor [1/s]
Z arbitrary second order tensor Z [1]

Greek

δ_r unit vector in the radial direction [1]

k-symbols

dk/dt rate of the unit normal vector [1/s]
$d\nabla_s k/dt$ rate of the interfacial gradient operator [m^{-1}/s]

Time and spatial derivatives

$\overset{\circ}{\mathbf{b}}$ co-rotational Zaremba-Jaumann derivative [m^{-1}/s]
$\dot{\mathbf{b}}$ rate of change of curvature tensor [m^{-1}/s]
$\dot{D}$ time derivative of the deviatoric curvature [m^{-1}/s]
$\dot{H}$ time derivative of the mean surface curvature [m^{-1}/s]