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A simple computational homogenization method for structures made of linear heterogeneous viscoelastic materials

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ABSTRACT

In this paper, a numerical multiscale method is proposed for computing the response of structures made of linearly non-aging viscoelastic and highly heterogeneous materials. In contrast with most of the approaches reported in the literature, the present one operates directly in the time domain and avoids both defining macroscopic internal variables and concurrent computations at micro and macro scales. The macroscopic constitutive law takes the form of a convolution integral containing an effective relaxation tensor. To numerically identify this tensor, a representative volume element (RVE) for the microstructure is first chosen. Relaxation tests are then numerically performed on the RVE. Correspondingly, the components of the effective relaxation tensor are determined and stored for different snapshots in time. At the macroscopic scale, a continuous representation of the effective relaxation tensor is obtained in the time domain by interpolating the data with the help of spline functions. The convolution integral characterizing the time-dependent macroscopic stress–strain relation is evaluated numerically. Arbitrary local linear viscoelastic laws and microstructure morphologies can be dealt with. Implicit algorithms are provided to compute the time-dependent response of a structure at the macroscopic scale by the finite element method. Accuracy and efficiency of the proposed approach are demonstrated through 2D and 3D numerical examples and applied to estimate the creep of structures made of concrete.

1. Introduction

Designing composite materials with tuned viscoelastic properties is a major concern in engineering. In polymer composites, damping properties can be desired in tandem with strength properties. In concrete structures, reducing the magnitude of creep allows diminution of the associated damage [16]. The related experimental relaxation tests are extremely costly and can last for months or years. Progresses in the design of high performance concrete then require predictive models and simulation methods taking into account the microstructure of the material.

Analytical methods for the homogenization of linear viscoelastic media have been proposed since the works of Hashin [7,8], who exploited the correspondence principle between linear elasticity and viscoelasticity by mean of the Laplace transform. In the Laplace space, classical homogenization methods such as the self-consistent scheme [13,24,15,1,21,2] and Mori–Tanaka technique [25,6,18,5,3] can be applied. The main issue is then the inversion of the Laplace transform which, in most cases, need to be performed numerically (see e.g. [26,9,14]). Accuracy and computational costs of this numerical inversion are serious issues. When applied to homogenization, the restrictive assumptions underlying the analytical methods on the morphology and local constitutive laws prevent them from being applied to complex realistic microstructures. Then, numerical method must be employed to solve the microscale spatial equations. Some methodologies have been proposed. For example, in [17], the microscopic spatial equations are solved by the generalized cell method.

To overcome the limitations of approaches based on the Laplace transform, alternative numerical methods operating in the time domain have been suggested. Lahellec and Suquet [12] introduced a scheme in which the notion of macroscopic internal variables related to an effective viscous strain is involved. Their method is based on an incremental variational principle and the variational approach of Ponte Castañeda [19]. Ricaud and Masson [20] proposed a different way taking advantage of the Prony–Dirichlet series expansion in the internal variable formulation. Another possible methodology corresponds to a two-scale numerical procedure [11,4] where each integration point of the macroscopic structure is associated to a representative volume element and, at every time step, the macroscopic strains at each integration point are taken to be the boundary conditions for the relevant local problem. The numerical solution to this problem gives the effective stresses. These methodologies induce important computational costs due to the nested numerical solvers and the storage of internal variables,

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even though progresses have been made by means of parallel computing [4] or model reduction [28].

The purpose of this paper is to present an efficient and simple methodology to compute the effective time-dependent response of structures consisting of linearly viscoelastic heterogeneous materials and undergoing arbitrary loadings. The homogenized constitutive law of a linearly viscoelastic heterogeneous material takes the form of a convolution integral involving an effective relaxation tensor which cannot be in general determined analytically. One of the main steps of our approach is to numerically determine all the components of the effective relaxation tensor directly in the time domain. This is realized as follows: (i) a representative volume element (RVE) for the microstructure of the linearly viscoelastic heterogeneous material in question is chosen and subjected to appropriate relaxation test loadings; (ii) the overall time-dependent response of the RVE is computed by using some efficient algorithms (see e.g. [10,22,23]); (iii) the numerical results obtained at different time steps and stored during the previous preliminary computations are interpolated with some appropriate spline functions. Then, the convolution integral is evaluated numerically so as to yield the macroscopic stress–strain relation for the computation of structures.

Compared to existing approaches, the one elaborated in the present work offers the following advantages: (a) the method operates directly in the time domain and avoids the drawbacks of the techniques based on the Laplace transform; (b) the formulation needs not to introduce any macroscopic internal variables; (c) in contrast with the numerical methods using concurrent calculations at the microscopic and macroscopic scales, the data required to determine the effective constitutive laws can be calculated in a preliminary step, so that, once they are stored, structure calculations can be carried out without solving any new problems on the RVE (for a related work on nonlinear homogenization, see [27]); (d) the implementation of the proposed approach is simple and classical implicit time-stepping algorithms can be directly employed.

The paper is organized as follows. In the next section, we briefly review the equations and algorithms for formulating and solving the local viscoelastic problem defined over an RVE. In Section 3 we present the methodology for sampling and interpolating the values of the effective relaxation tensor. Fully implicit algorithms are then detailed to compute the macroscopic structural response. In Section 4, we illustrate the proposed method and test its accuracy and efficiency through different 2D and 3D examples, with applications to the analysis of structures made of concrete.

2. Microscopic viscoelastic problem

We consider a structure made of a heterogeneous material whose phases are linearly and non-aging viscoelastic. We assume that the microstructure is defined by a representative volume element occupying a domain Ω, as depicted in Fig. 1(b). The sub-domains occupied by the different phases are $\Omega^{(r)}$ $(r = 1,2,\ldots,R)$ such that $\Omega = \bigcup_{r=1}^{R}\Omega^{(r)}$. In this section, we review equations and algorithms for solving linear homogeneous viscoelastic problems. We focus on the generalized Maxwell model which, with an infinite number of branches, is the most general one for linear viscoelasticity.

2.1. Linear viscoelasticity: generalized Maxwell model

2.1.1. 1D formulation

A linearly viscoelastic material can be characterized by a stress–strain relationship in the form of a convolution integral:

$$\sigma(t) = \int_{-\infty}^{t} G(t-s)\frac{d\varepsilon(s)}{ds}ds, \tag{1}$$

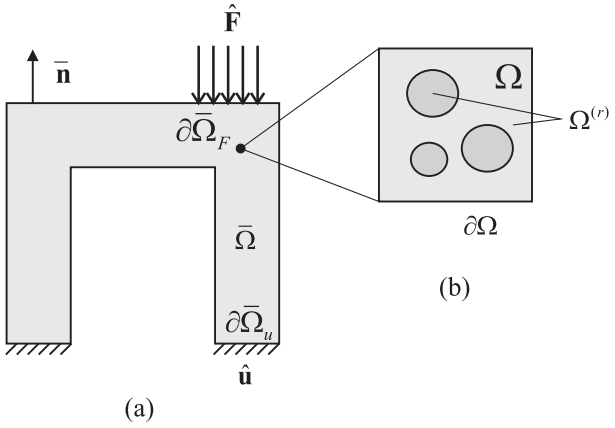

Fig. 1. (a) Macroscopic structure and (b) representative volume element.

where $G(t)$ is the relaxation modulus function. The integral in (1) is a Riemann–Stieltjes integral. It will be convenient to consider only time-dependent stress $\sigma(t)$ and strain $\varepsilon(t)$ which are null for $t < 0$, and which may have jump discontinuities at $t = 0$. In this case, we write (1) in the form

$$\sigma(t) = \int_0^t G(t-s)\frac{d\varepsilon(s)}{ds}ds + G(t)\varepsilon(0). \tag{2}$$

We consider the generalized Maxwell model as depicted in Fig. 2. The corresponding relaxation modulus function is given by (see details in Appendix A):

$$G(t) = E_\infty + \sum_{i=1}^{N} E_i \exp(-t/\tau_i), \tag{3}$$

where N is the number of parallel viscoelastic elements, E_∞, E_i are Young's moduli as shown in Fig. 2, and τ_i are the relaxation times of the parallel viscoelastic elements. Substituting (3) into (2), the total stress is given by

$$\sigma(t) = \int_0^t \dot{\sigma}_\infty(s)ds + \sum_{i=1}^{N}\int_0^t \gamma_i \exp(-(t-s)/\tau_i)\dot{\sigma}_\infty(s)ds + \left(1 + \sum_{i=1}^{N}\gamma_i \exp(-t/\tau_i)\right)\sigma_\infty(0), \tag{4}$$

where $\sigma_\infty(t) = E_\infty\varepsilon(t)$ and $\gamma_i = E_i/E_\infty$. By introducing

$$q_i = \int_0^t \gamma_i \exp[-(t-s)]/\tau_i\dot{\sigma}_\infty(s)ds \tag{5}$$

as internal stress variables, we finally obtain

$$\sigma(t) = \sum_{i=1}^{N} q_i + \sum_{i=1}^{N}\gamma_i \exp(-t/\tau_i)\sigma_\infty(0) + \sigma_\infty(t). \tag{6}$$

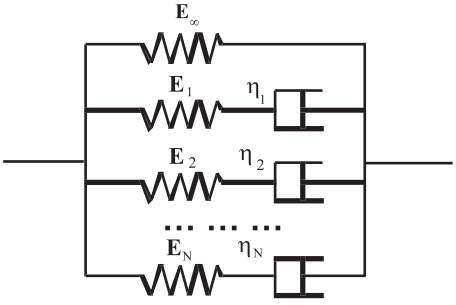

Fig. 2. Schematic representation of the generalized Maxwell model.