Comput. Methods Appl. Mech. Engrg. 200 (2011) 3169–3182

Contents lists available at SciVerse ScienceDirect

Comput. Methods Appl. Mech. Engrg.

journal homepage: www.elsevier.com/locate/cma

Uncertainty quantification of MEMS using a data-dependent adaptive stochastic collocation method

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ARTICLE INFO

Article history:
Received 23 December 2010
Received in revised form 22 June 2011
Accepted 23 June 2011
Available online 1 July 2011

Keywords:
Uncertainty quantification
Hybrid electrothermomechanical (ETM) actuation
Microelectromechanical systems (MEMS)
Stochastic collocation
Adaptive analysis

ABSTRACT

This paper presents a unified framework for uncertainty quantification (UQ) in microelectromechanical systems (MEMS). The goal is to model uncertainties in the input parameters of micromechanical devices and to quantify their effect on the final performance of the device. We consider different electromechanical actuators that operate using a combination of electrostatic and electrothermal modes of actuation, for which high-fidelity numerical models have been developed. We use a data-driven framework to generate stochastic models based on experimentally observed uncertainties in geometric and material parameters. Since we are primarily interested in quantifying the statistics of the output parameters of interest, we develop an adaptive refinement strategy to efficiently propagate the uncertainty through the device model, in order to obtain quantities like the mean and the variance of the stochastic solution with minimal computational effort. We demonstrate the efficacy of this framework by performing UQ in some examples of electrostatic and electrothermomechanical microactuators. We also validate the method by comparing our results with experimentally determined uncertainties in an electrostatic microswitch. We show how our framework results in the accurate computation of uncertainties in micromechanical systems with lower computational effort.

1. Introduction

Uncertainty quantification has become an important component in the design of most engineering systems and is widely used to predict the performance of a device, given a random manufacturing or operating environment. It is especially important in areas like micromachining, where manufacturing tolerances are much worse than what one might design for. Micromechanical devices are typically fashioned on wafers, with each wafer supporting anywhere from a few hundred to a few thousand devices. Ideally, the design of a typical microelectromechanical system (MEMS) must be robust enough to tolerate deviations in fabrication processes not only from one wafer to the next, but also spatially across a single wafer. However, due to practical limitations, it is not always possible to tightly control tolerances and this results in very different behavior, going from one device to the next. Although several advances have been made in the development of high-fidelity numerical methods to model microsystems [1–6], there is a pressing need to augment the capability of these methods to handle the uncertainties that are encountered in a realistic device model. In some cases, the undesirable effects caused by certain uncertainties can be mitigated by overcompensating other parameters, e.g. by increasing the voltage applied on an RF-MEM switch, one can ensure that pull-in will occur even in the presence of variations in the mechanical stiffness of the switch electrodes. However, this practice results in poor, inefficient designs and may even affect the lifetime of the device, e.g. the lifetime of an RF-MEM switch is shown to decrease exponentially with the voltage applied across it [7], due to phenomena like dielectric charging. In this paper, we develop a framework to accurately quantify the effect of uncertainties in MEMS in order to improve the design of such devices. A first step towards tackling this problem is to increase our understanding of the random processes at play by modeling the uncertainty and accurately calculating its statistics of interest.

In order to perform uncertainty analysis in MEMS, we face two important challenges. The first challenge is to identify and model the sources of uncertainty that affect a device of our interest. For most systems, the nature of uncertainty in the input parameters is usually unknown and is approximated using some standard distribution [8]. However, in the case of parameters encountered in micromechanical devices, it may not always be possible to describe the uncertainty using a standard distribution. Agarwal and Aluru [9] proposed a data-driven framework that generates a probability distribution for input parameters by estimating the uncertainty using actual experimental data. A data-driven framework does not place any assumptions on the nature of uncertainty and is extensible to any device parameter for which sufficient experimental data is available.

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doi:10.1016/j.cma.2011.06.010

The second challenge is to propagate the stochastic model through the system to understand how the uncertainties propagate into output quantities of interest. Traditionally, the engineering answer to this problem is to construct an elaborate *design of experiments* (DOE) that sufficiently replicates the gamut of variations that might typically be encountered. However, in the case of micromechanical devices, such an approach might turn out to be quite expensive in terms of time and effort. For particular types of uncertainty, it may not even be possible to come up with a comprehensive DOE procedure that captures the randomness with sufficient accuracy. With the advent of numerical solvers that accurately model complex multiphysics phenomena in devices, it is now possible to replicate the DOE on a computer without compromising on the accuracy in the results. There are many methods available in the literature, that are tailored towards performing this uncertainty quantification efficiently. Sampling-based methods like the Monte Carlo method have been used as the basis for implementing reliability-based design optimization techniques [10–12] to achieve robust designs in microsystems. Recently, there has also been a focus on non-statistical methods based on the Galerkin formulation, such that those that employ wavelet-based basis functions [13,14] and generalized polynomial chaos [8,15,16]. Stochastic collocation [17,18] is another method that has recently gained popularity. This method uses the idea of collocation, where the stochastic solution is approximated by interpolating the values of the solution obtained at specific collocation points in the parameter space. Modifications of this method have been suggested, where adaptive refinement of the collocation grid [19,20] is used to reduce the computational cost of constructing the interpolant. To perform UQ in realistic MEM devices, it is important to keep in mind that the particular method that we employ should also be flexible enough to accommodate any arbitrary stochastic model that is generated by the data-driven framework. It is also highly desirable to be able to propagate the uncertainty with minimal computational effort.

The goal of this paper is to present a unified framework for the quantification of uncertainty in MEMS. We are mainly interested in quantifying uncertainties by means of their statistical parameters, e.g. mean, variance, etc., which are the quantities that are most useful from a practical designer's point of view. Hence, we shall limit our focus to methods that perform the task of estimating these statistics efficiently with minimal computational effort, while maintaining accuracy. Our goal is to develop a generic model to describe uncertainties commonly encountered in microsystems and to quantify the effect of these uncertainties on device performance.

The paper is organized as follows: Section 2 describes the physical models governing the behaviour of the microactuator devices that we consider. These models are used to numerically simulate the device in order to perform uncertainty quantification. Section 3 then examines the sources of randomness in these devices and explains the generation of stochastic models that describe these uncertainties. It also introduces several methods for the propagation of these uncertainties through the device model, which are used to characterize the variation in output parameters of interest. In this context, it argues for the use of methods for the efficient estimation of statistics of the stochastic solution. All these methods are applied for the quantification of uncertainty in a few example micromechanical devices described in Section 4. We consider electromechanical actuators and describe how variations in their geometrical or material parameters affect their behaviour. Section 5 presents the concluding remarks.

2. Physical level modeling

We perform uncertainty quantification on the general class of electrothermomechanical (ETM) microactuators, which convert electrical energy to mechanical force or displacement. Based on the actuation mechanism, most of these actuators can be classified as either electrostatic actuators or electrothermal actuators. Recently, it has been shown that these two modes of actuation can also be integrated in a single device [21]. This hybrid ETM device operates with greater efficiency, producing larger displacements using the same operational voltage. Furthermore, it can be shown that the hybrid ETM model describes a more general class of devices, of which electrothermal and electrostatic actuators are special cases. We choose this model for demonstrating UQ due to the highly coupled interaction between electrical, thermal and mechanical fields, making it an ideal device for studying the propagation of uncertainties.

2.1. Hybrid ETM actuators

Hybrid ETM actuators integrate electrostatic and electrothermal modes of actuation in a single device. A typical device comprises a movable electrode placed next to a fixed ground plate, as shown in Fig. 1. The movable electrode has a double beam shape similar to a bent-beam electrothermal microactuator and is equipped with two electrical terminals. Potential differences may be applied across the terminals of the movable electrode as well as between the terminals of the movable and fixed electrodes. This results in the flow of current within the movable electrode, which causes Joule heating and consequently, electrothermal expansion. Due to the characteristic shape of this electrode, the constrained expansion forces it to bend towards the fixed electrode. The potential difference between the fixed and moving electrodes also creates an electrostatic field between them and corresponding electrostatic traction on the boundaries. Using a combination of these two forces, it is possible to achieve a larger displacement for the same applied potential difference than that obtained in similar actuators that employ either pure electrostatic or electrothermal actuation. In this section, we describe a physical model that governs the displacement of the actuator in terms of the device parameters. We present only a brief outline of the model here; a more complete description of the model is given in [21].

We model the problem using the two dimensional domain as shown in Fig. 1. In order to solve for the displacement in the actuator, we need to obtain a self-consistent solution between the coupled mechanical, thermal and electrical fields in the problem domain. Since we are interested in the steady-state behaviour of the device, we ignore all transient effects and seek the equilibrium solution of these fields. We also note that the mechanical displacement in the movable electrode causes the problem domain to deform and necessitates that the equations be expressed and solved in the deformed configuration for accurate results. This can be easily achieved by transforming all the equations from the deformed configuration to the undeformed reference domain (denoted by Ω_1, Ω_2 and $\overline{\Omega}$) corresponding to the initial state. This mapping is adequately described by the deformation gradient, $\boldsymbol{F}$, in the

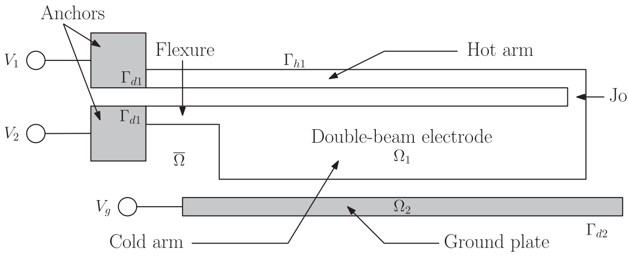

Fig. 1. Problem domain used to model a typical hybrid ETM actuator.