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Shell Finite Cell Method: A high order fictitious domain approach for thin-walled structures

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ABSTRACT

This article presents a generalization of the recently proposed Finite Cell Method to thin-walled structures. This approach uses a combination of well known Fictitious Domain Methods with high order hierarchical Ansatz spaces known from the *p*-version of the Finite Element Method. Whereas the original concept embeds a three-dimensional structure in a simple domain being meshed into a grid of cube shaped cells, the extension presented in this paper applies the fictitious domain approach to a two-dimensional master domain defined in the parameter plane of the geometric model. Implementation details are discussed and numerical benchmark problems show the high accuracy and computational efficiency of the new approach. It is also remarked, that the present approach can easily be carried over to isogeometric analysis, opening an attractive possibility to simulate trimmed NURBS-surfaces.

1. Introduction

Since the early days of the Finite Element Method a lot of research has been devoted to get around the necessity to model the boundary of a domain of computation exactly and thus to ease the effort of mesh generation. These approaches, each with their individual mathematical or algorithmic features, are known as Fictitious Domain Methods [1–4], Embedding Domain Methods [5–7] or Immersed Boundary Methods [8,9]. Also many variants of meshfree or meshless methods [10–13] and Extended Finite Element Methods [14,15] or Level Set Methods [16,17] can be seen in this category. An overview of the huge body of literature is given, for example, in [18–20]. Common to all these approaches is the embedding of the domain of computation Ω into an extended domain Ω_e, typically with a geometrically simple shape, which can easily be discretized in a structured mesh or even a Cartesian grid for computation. This mesh or grid does not necessarily follow the boundary of the original domain Ω.

The Finite Cell Method (FCM), recently proposed in [21,22], uses some of the basic concepts of fictitious domain approaches, but extends them to high order Ansatz spaces known from the *p*-version of the Finite Element Method. Dividing Ω_e into a (regular) grid of *cells* and applying an Ansatz with higher order polynomials, the FCM has been investigated for linear elasticity in 2D and 3D [21,22], for topology optimization problems [23,24], geometrically nonlinear problems [25,26] and for problems in elastoplasticity [27]. Adaptive schemes with hierarchical spline base functions have been developed [28] and a very fast implementation using pre-integrated stiffness matrices has been used for interactive 3D-simulation in a computational steering system with application in biomedical engineeering [29,30]. The Finite Cell Method proves in all these cases to have significant advantages over classical Finite Element Methods or over low order fictitious domain approaches. In addition to the (very important) practical benefit of relieving from the necessity of mesh generation it shows astonishing accuracy and superior computational efficiency.

Yet, one significant limitation of the FCM derives from the requirement that features of the domain of computation Ω (e.g. holes, girders) should not have dimensions which are significantly smaller than the size of the computational cells. This makes it for the original method inefficient or practically even impossible to simulate thin-walled structures like beams, frames or shells. In this paper the method is, therefore, extended to general three-dimensional thin-walled structures, separating the description of the structural geometry from the fictitious domain approach. Whereas the 'cells' with their higher order Ansatz are defined by a Cartesian grid in parameter space, the structure in physical space is covered by a geometry-aligned mesh of generalized hexahedra with a shape being defined by the geometric model. This model can be based on B-splines, NURBS or any other description suitable in geometric modeling. From a conceptual point of view this

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approach has several similarities to the Isogeometric Element Method [31] and opens a very simple and efficient way to use 'trimmed patches' as basic geometric objects.

This paper is organized as follows: In the next section we give a short summary of the basic ideas of the FCM. Section 4 will extend the approach to three-dimensional thin-walled structures. In Section 5 we will first demonstrate the high accuracy and efficiency of the method on a modification of a classical shell benchmark problem and finally show a geometrically complex example of a structural vibration analysis.

2. The Finite Cell Method: basic formulations

For completeness of this paper we briefly summarize the basic concepts of the Finite Cell Method, closely following the description in [21,22]. We restrict the formulation to linear elasticity in 3D (for clarity, all figures are given in 2D), yet remark, that an extension to nonlinear problems like elastoplasticity is possible, see [27].

Let us assume on a three-dimensional physical domain Ω a problem of linear elasticity, described by the weak form of equilibrium as

$$\mathcal{B}(\mathbf{u},\mathbf{v}) = \mathcal{F}(\mathbf{v}), \tag{1}$$

where the bilinear part is

$$\mathcal{B}(\mathbf{u},\mathbf{v}) = \int_{\Omega} [\mathbf{L}\ \mathbf{v}]^T \mathbf{C} [\mathbf{L}\mathbf{u}]\, d\Omega \tag{2}$$

in which $\mathbf{u}$ is the displacement, $\mathbf{v}$ is the test function, $\mathbf{L}$ is the standard strain–displacement operator and $\mathbf{C}$ is the elasticity matrix. We assume Dirichlet boundary conditions $\bar{\mathbf{u}}$ along Γ_D and a Neumann boundary Γ_N with prescribed tractions, $\partial\Omega = \Gamma_D \cup \Gamma_N$, and $\Gamma_D \cap \Gamma_N = \emptyset$. The linear functional

$$\mathcal{F}(\mathbf{v}) = \int_{\Omega} \mathbf{v}^T \mathbf{f}\, d\Omega + \int_{\Gamma_N} \mathbf{v}^T \bar{\mathbf{t}}\, d\Gamma \tag{3}$$

takes the volume loads $\mathbf{f}$ and prescribed tractions $\bar{\mathbf{t}}$ into account.

The original physical domain can now be embedded in the domain Ω_e with the boundary $\partial\Omega_e$ (Fig. 1). Following Neittaanmäki and Tiba [32], the displacement variable is extended as:

$$\mathbf{u} = \begin{cases} \mathbf{u}^1 & \text{in } \Omega \\ \mathbf{u}^2 & \text{in } \Omega_e \setminus \Omega \end{cases} \tag{4}$$

and continuity of displacements and tractions is assumed at the interface between Ω and $\Omega_e \backslash \Omega$:

Boundary conditions are set for $\partial\Omega_e$

$$\begin{aligned} \bar{\mathbf{u}} &= \mathbf{0} \quad \text{on } \Gamma_{e,D}, \\ \bar{\mathbf{t}} &= \mathbf{0} \quad \text{on } \Gamma_{e,N}. \end{aligned} \tag{5}$$

in which $\Gamma_{e,D}$ and $\Gamma_{e,N}$ are the Dirichlet and Neumann boundaries of Ω_e respectively, $\partial\Omega_e = \Gamma_{e,D} \cup \Gamma_{e,N}$, and $\Gamma_{e,D} \cap \Gamma_{e,N} = \emptyset$. The weak form of the equilibrium equation for the embedding domain Ω_e is given as

$$\mathcal{B}_e^\alpha(\mathbf{u},\mathbf{v}) = \mathcal{F}_e^\alpha(\mathbf{v}), \tag{6}$$

where the bilinear form is

$$\mathcal{B}_e^\alpha(\mathbf{u},\mathbf{v}) = \int_{\Omega_e} [\mathbf{L}\ \mathbf{v}]^T \mathbf{C}_e^\alpha [\mathbf{L}\ \mathbf{u}]\, d\Omega \tag{7}$$

in which

$$\mathbf{C}_e^\alpha = \alpha \mathbf{C} \tag{8}$$

is the elasticity matrix of the embedding domain, with

$$\alpha(\mathbf{x}) = 1.0 \quad \forall \mathbf{x} \in \Omega, \tag{9}$$

$$0.0 \leqslant \alpha(\mathbf{x}) \leqslant 1.0 \quad \forall \mathbf{x} \in \Omega_e \setminus \Omega. \tag{10}$$

In the case of $\alpha(\mathbf{x}) = 0$ for $\mathbf{x} \in \Omega_e \backslash \Omega$ the bilinear functional turns to

$$\begin{aligned} \mathcal{B}_e^\alpha(\mathbf{u},\mathbf{v}) &= \int_{\Omega_e} [\mathbf{L}\ \mathbf{v}]^T \alpha \mathbf{C} [\mathbf{L}\ \mathbf{u}]\, d\Omega \\ &= \int_{\Omega} [\mathbf{L}\ \mathbf{v}]^T \mathbf{C} [\mathbf{L}\ \mathbf{u}]\, d\Omega + \int_{\Omega_e \backslash \Omega} [\mathbf{L}\ \mathbf{v}]^T \mathbf{0} [\mathbf{L}\ \mathbf{u}]\, d\Omega = \mathcal{B}(\mathbf{u},\mathbf{v}). \end{aligned} \tag{11}$$

The linear functional

$$\mathcal{F}_e^\alpha(\mathbf{v}) = \int_{\Omega_e} \mathbf{v}^T \alpha \mathbf{f}\, d\Omega + \int_{\Gamma_N} \mathbf{v}^T \bar{\mathbf{t}}\, d\Gamma + \int_{\Gamma_{e,N}} \mathbf{v}^T \bar{\mathbf{t}}\, d\Gamma \tag{12}$$

considers the volume loads $\mathbf{f}$, prescribed traction along Γ_N *interior* to Ω_e and prescribed traction at the boundary of the embedding domain. Due to Eq. $(5)_2$, the last term in (12) can be assumed 0.

As for the bilinear function it can immediately be seen that for $\alpha(\mathbf{x}) = 0$ where $\mathbf{x} \in \Omega_e \backslash \Omega$ the extended load functional (12) equals (3). Furthermore, for $\alpha \to 0$ in $\Omega_e \backslash \Omega$ the solution $\mathbf{u}_e^\alpha$ of (6) converges towards the solution $\mathbf{u}$ of (1).

The embedding domain is now discretized in a Cartesian grid which is independent of the original domain. These "finite elements" of the embedding domain do not necessarily fulfill the usual geometric properties of elements for the original domain Ω, as they may be intersected by $\partial\Omega$. To distinguish them from classical elements they are called *finite cells*. Fig. 2 illustrates the situation for a two-dimensional setting.

At the discretized level, the weak form (7) turns into

$$\mathcal{B}_e^\alpha(\mathbf{u},\mathbf{v}) = \sum_{c=1}^{n_c} \int_{\Omega_c} [\mathbf{L}\ \mathbf{v}]^T \alpha \mathbf{C} [\mathbf{L}\ \mathbf{u}]\, d\Omega. \tag{13}$$

In each cell the displacement variable is approximated as

$$\mathbf{u} = \mathbf{N}\mathbf{U}^\mathbf{c}, \tag{14}$$

where $\mathbf{N}$ denotes the matrix of shape functions and $\mathbf{U^c}$ is the vector of unknowns associated to the cell $\mathbf{c}$ under consideration. As usual, the product $\mathbf{LN}$ is denoted as strain–displacement matrix $\mathbf{B}$. Our implementation, [33], uses hexahedrals with hierarchical shape

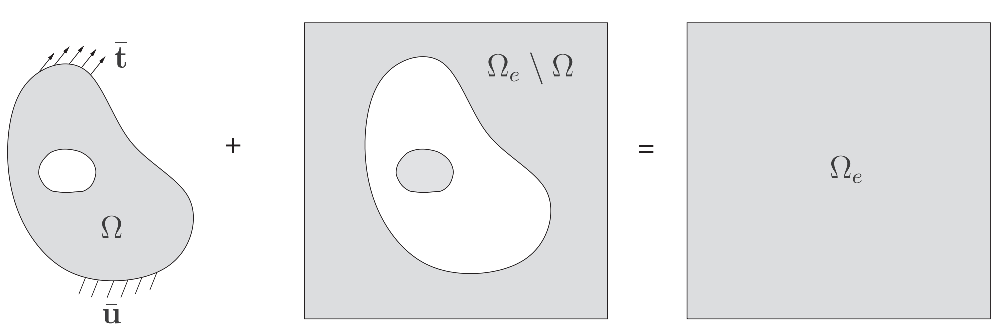

Fig. 1. The domain Ω is embedded in Ω_e.