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A new approach to solid modeling with trivariate T-splines based on mesh optimization

J.M. Escobar [a], J.M. Cascón [b], E. Rodríguez [a], R. Montenegro [a,*]

[a] University of Las Palmas de Gran Canaria, University Institute for Intelligent Systems and Numerical Applications in Engineering (SIANI), Spain
[b] University of Salamanca, Department of Economics and History of Economics, Faculty of Economics and Management, Spain

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ABSTRACT

We present a new method to construct a trivariate T-spline representation of complex genus-zero solids for the application of isogeometric analysis. The proposed technique only demands a surface triangulation of the solid as input data. The key of this method lies in obtaining a volumetric parameterization between the solid and the parametric domain, the unitary cube. To do that, an adaptive tetrahedral mesh of the parametric domain is isomorphically transformed onto the solid by applying a mesh untangling and smoothing procedure. The control points of the trivariate T-spline are calculated by imposing the interpolation conditions on points sited both on the inner and on the surface of the solid. The distribution of the interpolating points is adapted to the singularities of the domain in order to preserve the features of the surface triangulation.

1. Introduction

CAD models usually define only the boundary of a solid, but the application of isogeometric analysis [2,3,10] requires a fully volumetric representation. An open problem in the context of isogeometric analysis is how to generate a trivariate spline represen tation of a solid starting from the CAD description of its boundary. As it is pointed by Cotrell et al. in Ref. [10], "the most significant challenge facing isogeometric analysis is developing three-dimensional spline parameterizations from surfaces".

There are only a few works addressing this problem, and they all have in common the use of harmonic functions to establish the volumetric parameterization [19,21–23,30].

For example, Li et al. [19] construct a harmonic volumetric mapping through a meshless procedure by using a boundary method. The algorithm can be applied to any genus data but it is complex and requires placing some source and collocation points on an offset surface. Optimal results of source positions are unknown, and in practice they are chosen in a trial-and-error manner or with the help of human experience. Therefore, the problem is ill-conditioned and regular system solvers often fail.

Martin et al. [22] and Martin and Cohen [23] present a methodology based on discrete harmonic functions to parameterize a solid. They solve several Laplace's equations, first on the surface and then on the complete 3-D domain with FEM, and use a Laplacian smoothing to remove irregularities. During the process, new vertices are inserted in the mesh and retriangulations (in 2-D and 3-D) are applied in order to introduce the new vertex set in the mesh. The user has to make an initial choice of two critical points to establish the surface parameterization and to fix a seed for generating the skeleton. The parameterization has degeneracy along the skeleton. The extension to genus greater than zero [23] requires finding suitable midsurfaces.

We propose a different approach in which the volumetric parameterization is accomplished by transforming a tetrahedral mesh from the parametric domain to the physical domain. This is a special feature of our procedure; we do not have to give the tetrahedral mesh of the solid as input, as it is a result of the parameterization process. Another characteristic of our work is that we use an interpolation scheme to fit a trivariate B-spline to the data, instead of an approximation, as other authors do. This performs a more accurate adaptation of the T-spline to the input data.

One of the main drawbacks of NURBS (see for example [26]) is that they are defined on a parametric space with a tensor product structure, making the representation of detailed local features inefficient. This problem is solved by the T-splines, a generalization of NURBS conceived by Sederberg et al. [27] that enables the local refinement. The T-splines are a set of functions defined on a T-mesh, a tiling of a rectangular prism in $\mathbb{R}^3$ allowing T-junctions (see [2,27]).

In this paper we present a new method for constructing volumetric T-meshes of genus-zero solids whose boundaries are defined

* Corresponding author. Tel.: +34 928457400; fax: +34 928457401.
E-mail address: rafa@dma.ulpgc.es (R. Montenegro).

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by surface triangulations. Our procedure can be summarized in two stages. In the first one, a volumetric parameterization of the solid is developed. Broadly speaking, we can consider that the construction of a volumetric parameterization is a process in which an adaptive tetrahedral mesh, initially defined in the unitary cube $\mathcal{C} = [0,1]^3$, is deformed until it achieves the shape of the solid (the physical domain). This deformation only affects the positions of the nodes, that is, there is not any change in their connectivities: we say that both meshes are isomorphic. Given that a point is fully determined by the barycentric coordinates relative to the tetrahedron in which it is contained, we can define a one-to-one mapping between $\mathcal{C}$ and the solid assuming that the barycentric coordinates are the same in both spaces.

In the second stage, the modeling of the solid by trivariate T-splines is carried out. The control points of the T-splines are calculated enforcing the T-splines to verify the interpolation conditions. Here is where the volumetric parametrization plays its part, mapping the interpolation points from the parametric domain, the T-mesh, onto the solid. In our case, the T-mesh is an octree partition of $\mathcal{C}$ with a similar resolution than the tetrahedral mesh defined in $\mathcal{C}$.

Our technique is simple and it automatically produces a T-spline adapted to the geometry with a low computational complexity and low user intervention. As in other methods, our parameterization can introduce some distortion, especially along the cube edges.

The paper is organized as follows: In the next section we describe the main steps to parameterize a genus-zero solid onto a cube. Some parts of this section are taken from our previous works on mesh untangling and smoothing and the meccano method [7,8,11,24,25], but they have been adapted to the requirements of the present work. The representation of the solid by means of trivariate T-splines is developed in Section 3. In Section 4 we show a test problem and several applications that highlight the ability of our method for modeling complex objects. Finally, in Section 5 we present the conclusions and set out some challenges.

2. Volumetric parameterization

2.1. Boundary mapping

The first step to construct a volumetric parameterization consists of establishing a bijective correspondence between the boundary of the cube and the solid. To do that, the given surface triangulation of the solid, $\mathcal{T}_S$, is divided in six patches or *connected subtriangulations*, $\mathcal{T}_S^i$ $(i = 1,2,\ldots,6)$, having the same connectivities as the cube faces. Specifically, if we consider that each subtriangulation corresponds to a vertex of a graph and two vertices of the graph are connected if their corresponding subtriangulations have at least a common edge, then, the graphs corresponding to the solid and the graph of the cube must be isomorphic (see [8,25] for details).

Once $\mathcal{T}_S$ is decomposed into six patches, we map each $\mathcal{T}_S^i$ to the corresponding cube face by using the parameterization of surface triangulations proposed by Floater in [13,14]. This is a well-known method to transform a surface triangulation onto a plane triangulation defined in a convex domain, that is, the cube faces in our case. Many and more recent alternative solutions have been proposed to solve the surface parameterization (see for example the surveys [15,16]), but in most of them the plane triangulation is not defined in a convex set, which is a restriction for us. Thus, if τ_F^i is the resulting triangulation on the ith face of the cube, the parameterization $\Pi_F^i : \tau_F^i \to \mathcal{T}_S^i$ is a piece-wise linear function that maps a point p inside triangle $T \in \tau_F^i$ onto a point q belonging to triangle $\Pi_F^i(T) \in \mathcal{T}_S^i$ with identical barycentric coordinates.

In order to ensure the compatibility of $\left\{\Pi_F^i\right\}_{i=1}^6$, the boundary nodes of $\{\tau_F^i\}_{i=1}^6$ must coincide on common cube edges. The six transformations $\left\{\Pi_F^i\right\}_{i=1}^6$ define a global parameterization between $\tau_F = \bigcup_{i=1}^6 \tau_F^i$ and $\mathcal{T}_S$ given by

$$\Pi_F : \tau_F \to \mathcal{T}_S. \tag{1}$$

The parameterization Π_F is used in the following step of the algorithm to map a new triangulation defined over the boundary of $\mathcal{C}$ onto the boundary of the solid.

2.2. Generation of an adapted tetrahedral mesh of the cube

Let consider $\mathcal{C}_K$ is a tetrahedral mesh of $\mathcal{C}$ resulting after applying several local bisections of the Kossaczky algorithm [18] to an initial mesh formed by six tetrahedra (see Fig. 1(a)). Three consecutive global bisections are presented in Fig. 1(b)–(d). The mesh of Fig. 1(d) contains eight cubes similar to the one shown in Fig. 1(a). Therefore, the successive refinement of this mesh produces similar tetrahedra to those of Fig. 1(a)–(c).

If $\tau_K = \partial\mathcal{C}_K$ is the new triangulation defined on the boundary of $\mathcal{C}$, then we define a new parameterization

$$\Pi_K : \tau_K \to \mathcal{T}_S^*, \tag{2}$$

where $\mathcal{T}_S^*$ is the surface triangulation obtained after Π_F-mapping the nodes of τ_K. The points of τ_K are mapped to $\mathcal{T}_S^*$ by preserving their barycentric coordinates. Note that $\mathcal{T}_S^*$ is an approximation of $\mathcal{T}_S$. In order to improve this approximation we must refine the tetrahedra of $\mathcal{C}_K$ in contact with the surface of the cube in such a way that the *distance* between $\mathcal{T}_S^*$ and $\mathcal{T}_S$ decreases until reaching a prescribed tolerance ε. The concept of *distance* between two triangulations can be defined and implemented in several ways. In our case, it is as follows:

Let T be a triangle of τ_K, where a, b and c are their vertices and let $p_k \in \{p_i\}_{i=1}^{N_q}$ be a Gauss quadrature point of T, then, the distance, $d(T)$, between $\Pi_K(T)$ and the underlaying triangulation $\mathcal{T}_S$ is defined as the maximum of the volumes of the tetrahedra formed by $\Pi_F(a)$, $\Pi_F(b)$, $\Pi_F(c)$ and $\Pi_F(p_k)$. If we consider the distance between $\mathcal{T}_S^*$ and $\mathcal{T}_S$ as the maximum of all $d(T)$, the local refinement stops when $d(T) < \epsilon$ for all $T \in \tau_K$. A more accurate approach based on Hausdorff distance can be found in [4].

Once the adapted tetrahedral mesh $\mathcal{C}_K$ has been constructed by using the proposed method, the nodes of τ_K are mapped to the

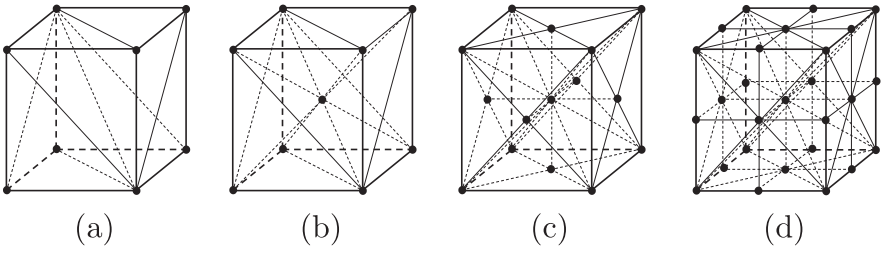

Fig. 1. Refinement of a cube by using Kossaczky's algorithm: (a) cube subdivision into six tetrahedra, (b) bisection of all tetrahedra by inserting a new node in the cube main diagonal, (c) new nodes in diagonals of cube faces and (d) global refinement with new nodes in cube edges.