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# A one field full discontinuous Galerkin method for Kirchhoff–Love shells applied to fracture mechanics

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ABSTRACT

In order to model fracture, the cohesive zone method can be coupled in a very efficient way with the finite element method. Nevertheless, there are some drawbacks with the classical insertion of cohesive elements. It is well known that, on one the hand, if these elements are present before fracture there is a modification of the structure stiffness, and that, on the other hand, their insertion during the simulation requires very complex implementation, especially with parallel codes. These drawbacks can be avoided by combining the cohesive method with the use of a discontinuous Galerkin formulation. In such a formulation, all the elements are discontinuous and the continuity is weakly ensured in a stable and consistent way by inserting extra terms on the boundary of elements. The recourse to interface elements allows to substitute them by cohesive elements at the onset of fracture.

The purpose of this paper is to develop this formulation for Kirchhoff–Love plates and shells. It is achieved by the establishment of a full DG formulation of shell combined with a cohesive model, which is adapted to the special thickness discretization of the shell formulation. In fact, this cohesive model is applied on resulting reduced stresses which are the basis of thin structures formulations. Finally, numerical examples demonstrate the efficiency of the method.



## 1. Introduction

When designing thin structures, tearing prediction remains a challenging modeling task. This increases the interest of fracture and rupture numerical models for thin bodies, which must be able to take into account a through-the-thickness fracture.

Among the different approaches in fracture mechanics, the cohesive zone concept pioneered by Dugdale [1] and Barenblatt [2] is based on a “Traction Separation Law” (TSL), which gives a relationship between the tension and the opening of the crack faces ($\Delta$), and can be easily combined with finite element (FE) methods [3–10]. In such an approach, cohesive elements, integrating the TSL, are inserted as interface elements between bulk elements. Unfortunately, as it is extensively discussed in [11,12], the two classical methods considered to introduce the cohesive elements suffer from severe limitations. On the one hand, an intrinsic cohesive law [4,6,9,13], for which cohesive elements are introduced at the beginning of the simulation, has to consider the pre-fracture stage by inserting an initial slope in the TSL (see Fig. 1). This initial slope must tend to infinity to ensure a correct wave propagation in the structure, which leads to some numerical problems [14]. On the other hand, an extrinsic cohesive law, where the cohesive elements are inserted on the fly during the simulation when a fracture criterion is reached [3,7,8,10], requires a very complex implementation [15–17] due to the inherent difficulty associated with propagating topological changes in the mesh. As a large number of degree of freedoms (dofs) is needed to obtain a convergence in a fracture problem [7], a parallel implementation can be required to perform large simulations in an admissible computational time, further complicating the implementation.

Some alternatives have been suggested [18–21] to compensate these different limitations, and one promising method, especially when considering 3D parallel simulations, is a new approach based on the combination of a full discontinuous Galerkin (DG) formulation and an extrinsic cohesive law. This method was pioneered by Mergheim et al. [22], by Radovitzky et al. [11,12], and by Prechtel et al. [23]. In this method, interface elements are inserted at the beginning of the simulation between discontinuous bulk elements to weakly ensure the compatibility condition in a stable and consistent manner. When a fracture criterion is reached, this interface element is then very easily replaced by a cohesive element. This method has recently been implemented for 3D elements by Radovitzky et al. in [12], where it was shown that this approach is scalable when parallelized and does not require the use of complex topological information. Moreover, this method can be

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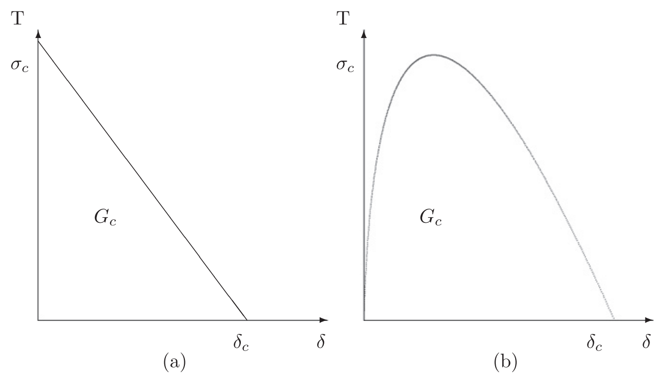


**Fig. 1.** Linearly decreasing monotonic (a) extrinsic and (b) intrinsic "Traction Separation Law".

implemented easily into existing FE codes. For completeness, let us note that the combination of cohesive law and discontinuous Galerkin method was also achieved by using a space–time discontinuous method [21].

This method can be seen as complementary to the XFEM method [24–29], among others. The XFEM formulation, pioneered by Moës and Belytschko [25] allows to take into account discontinuities inside elements, and is thus very appealing to model crack propagation. The most common XFEM approaches enrich nodes with the linear elastic fracture mechanics solution in order to extract the stress intensity factors, and to propagate a crack using the maximum hoop criterion. In the method based on the DG/extrinsic law combination, cracks can be initiated, merged, or propagate, without implementing explicitly crack propagation criteria, as the solution results naturally from the minimization of energy. Moreover as the DG/extrinsic law combination is well suited for dynamics and large computations in parallel, it makes a good alternative to XFEM for problems of fragmentation as for structures subjected to shock, blast, ... Although the DG/extrinsic law combination suffers from having cracks forced to follow the elements edges, and thus requires a fine mesh to capture the solution, as recently demonstrated by Duflot et al. [29] for 3D problems, the XFEM method also requires fine mesh around the crack tip to capture the crack path. On the implementation point of view, the DG/extrinsic law combination can easily be introduced in an existing parallel code, while the implementation of XFEM requires more attention. Indeed implementation of the XFEM method is challenging when the crack path is near a node, which requires re-meshing [29], when applying Dirichlet boundary conditions, ... Also, parallelizing the XFEM is not straightforward, which makes the DG/extrinsic law combination a good candidate to solve large scale problems.

In a recent paper, the authors extended the combination of a full discontinuous Galerkin (DG) formulation and an extrinsic cohesive law to Euler Bernoulli beams [30], and the present work wants to develop this formulation for Kirchhoff–Love shells fracture problems. Toward this end, a new one-field full discontinuous Galerkin (full-DG) discretization of the shell equations is obtained from the extension of the work of several authors [31–36] who have developed a $C^0$/DG formulation for thin bodies. In these works a DG method is used to weakly enforce the $C^1$ continuity required by high-order formulations of beams, plates and shells, which leads to displacement-field only methods (nodes have no degree of freedom of rotation). However to extend such a formulation to fracture, it is convenient to consider discontinuous test functions in order to insert interface elements. The study of the new resulting full-DG formulation shows that this formulation is consistent, stable (if stabilization parameters are larger than a mesh-independent constant) and that it converges in the $\mathbf{L}^2$ norm with the optimum rate.

In order to enhance this full DG formulation with fracture mechanics, the interface element should integrate the TSL upon the onset of fracture. However, as it is discussed in [30], the model of a "through-the-thickness fracture" is not straightforward for thin bodies since the thickness is modeled implicitly in the mesh discretization. To avoid the evaluation of the TSL at different points on the thickness, authors suggested in [30] to apply the cohesive principles directly to the resultant stresses (bending and membrane) in terms of the resultant openings (angle and mid-plane openings). The new Traction Separation Law is then defined in such a way that the model respects the energetic balance during the fracture process for any coupled bending-traction loadings. This model is extended here to linear shell elements by considering a combination of the different fracture modes.

The article is organized as follows. First, the governing equations of continuum mechanics of shells are recalled in Section 2. Afterward, in Section 3, these equations are formulated within a full-DG framework and the numerical properties are then, on the one hand, studied in an analytical way, and, on the other hand illustrated by a numerical example. In Section 4, an original cohesive zone model based on the resultant stresses is presented and is coupled with the full-DG formulation in order to take into account brittle fracture. The next section deals with some considerations about the implementation of the method. Section 6 presents several numerical applications of fracture testing to demonstrate the ability of the presented framework to simulate fracture problems of thin bodies. Finally some concluding remarks are drawn.

## 2. Continuum mechanics of thin bodies

The continuum mechanics of thin structures is well established and can be found in several Refs. [34,35,37,38]. For this reason, this section presents only the important results and the notations required to develop the full DG theory. More details can be found in the cited papers, and in particular the last two references use exactly the same notations and conventions as in this paper.

### *2.1. Kinematics of the shell*

A thin body can be described by considering its mid-surface section as a Cosserat plane $\mathcal{A}$ and a third coordinate, representing the thickness, belonging to the interval $[h_{\min}; h_{\max}]$. In the reference frame $\boldsymbol{E}_I$, this representation is written $\boldsymbol{\xi} = \sum_{I=1}^{3} \xi^I \boldsymbol{E}_I : \mathcal{A} \times [h_{\min}; h_{\max}] \rightarrow \mathbb{R}^3$. Hereinafter, a subscript will be used to refer to values expressed in the considered basis, while a superscript will be used to refer to values expressed in the conjugate basis. Of course, for the initial frame, $\boldsymbol{E}_I = \boldsymbol{E}^I$. Roman letters as a subscript or superscript substitute for integers between one and three, while Greek letters substitute for integers one or two. The representation of the body in the inertial frame is illustrated in Fig. 2. Using $\boldsymbol{\varphi}(\xi^1, \xi^2) : \mathcal{A} \rightarrow \mathbb{R}^3$ the mapping of the mid-surface and $\boldsymbol{t} : \mathcal{A} \rightarrow S^2 = \{\boldsymbol{t} \in \mathbb{R}^3|_{\|\boldsymbol{t}\|=1}\}$ the director of the mid-surface, with $S^2$ the unit sphere manifold, a configuration $\mathcal{S}$ of the shell is represented by the manifold of position $\boldsymbol{x}$, which is obtained by the mapping $\boldsymbol{\Phi} : \mathcal{A} \times [h_{\min}; h_{\max}] \rightarrow \mathcal{S}$,

$$\boldsymbol{x} = \boldsymbol{\Phi}(\xi^I) = \boldsymbol{\varphi}(\xi^\alpha) + \xi^3 \boldsymbol{t}(\xi^\alpha). \tag{1}$$

By convention, $\mathcal{S}$ refers to the current configuration of the shell, while the reference configuration $\mathcal{S}_0$ is obtained by the mapping $\boldsymbol{\Phi}_0$.

### *2.2. Governing equations of the linear shell*

The governing equations of a thin body are obtained by integrating on the thickness the equations of force and moment equilibrium, leading to