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Correlation analysis of non-probabilistic convex model and corresponding structural reliability technique

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ABSTRACT

Compared with the probability approach, the non-probabilistic convex model only requires a small amount of samples to obtain the variation bounds of the imprecise parameters, and whereby makes the reliability analysis very convenient and economical. In this paper, we attempt to propose and create a correlation analysis technique mathematically for the non-probabilistic convex model, and based on it develop an effective method to construct the multidimensional ellipsoids on the uncertainty. A marginal convex model is defined to describe the variation range of each uncertain parameter, and a covariance is defined to represent the correlation degree of two uncertain parameters. For a multidimensional problem, the covariance matrix and correlation matrix can be created through all marginal convex models and covariances, based on which the required ellipsoid on the uncertainty can be conveniently achieved. By combining the correlation analysis technique and the reliability index approach, a non-probabilistic reliability analysis method is also developed for uncertain structures. Six numerical examples are presented to demonstrate the effectiveness of the present method.

1. Introduction

Uncertainty widely exists in practical engineering problems, which are commonly related to material properties, loads, boundary conditions, etc. The probability model is most widely used to quantify the uncertainty, and whereby obtain the distributions of the structural responses based on the statistical techniques [e.g. 1–5]. The probability model has become a principal means to deal with the uncertainty and been successfully applied to varieties of industrial departments. In its treatments, a great amount of information is required to construct precise probability distributions for uncertain parameters, which, however, are not always available or sometimes very costly for practical problems. Thus some assumptions have to be made for probability distributions in many cases when using the probability model. Nevertheless, there are researches indicating that even a small deviation of the probability distributions from the real values may result an extremely large error of the uncertainty analysis [6].

Since entering the 1990s, Ben-Haim and Elishakoff [6–9] proposed a new kind of uncertainty analysis methodology based on the non-probabilistic convex model. In this method, the parameters' fluctuation is assumed to fall into a multidimensional ellipsoid or solid box which can be easily obtained only based on a small number of samples or just our engineering experience, and optimization problems are generally formulated to seek the most favorable and least favorable structural responses under the constraints defined by the convex sets. Due to its weak dependence on the sample amount, the convex model approach has been attracting growing attention, and some relevant analysis techniques have been developed. A comparison was made for the probability model and the convex model [9]. An "uncertain triangle" was employed to describe the relation between the three uncertainty analysis methods, namely probability, fuzzy sets and convex model [10]. A matrix perturbation method was developed to study the static responses and eigenvalues of structures based on the convex model [11]. A new interval analysis technique was proposed to calculate the static and dynamic responses of uncertain structures based on an improved first-order Taylor interval expansion [12]. An error estimation was suggested for interval and sub-interval analysis methods based on a second-order truncation model [13]. Applications of the convex model in engineering mechanics include non-linear buckling analysis of a column with uncertain initial imperfections [14], stability analysis of elastic bars on uncertain foundations [15], bound analysis of structural responses of beams [16], uncertain analysis in structural number determination in flexible pavement design [17], etc.

In recent years, convex model was also applied to the reliability analysis of uncertain structures and some works in this field have been published. By introducing the concept of traditional first order reliability method (FORM) into the problems with convex

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models, a non-probabilistic reliability index which represents a minimal distance in the standard convex space was defined [18,19] and an efficient solving algorithm was further formulated for this reliability index [20]. By putting the non-probabilistic reliability indexes as constraints, several reliability-based optimization design methods were developed [21–23]. Based on the order relation of convex sets, a nonlinear interval number programming with high efficiency was suggested for reliability design of uncertain structures [24,25]. By integrating the probability approach and the convex model, the hybrid reliability was also investigated [26,27].

In the above mentioned works, the uncertainty is generally quantified through a multidimensional ellipsoid, which represents the scattering extent of the uncertain parameters. In practice, for convex model and all relevant analysis techniques, this ellipsoid plays an extremely important role, just like the precise distributions of the parameters in probability analysis. Unfortunately, an effective method which can be used to construct the multidimensional ellipsoids efficiently and conveniently is still in absence. In almost all existing works, the ellipsoids on the uncertainty are just assumed to be given in advance, however, it should be pointed out that in practical applications no one seems to do it for us. Theoretically, the ellipsoid can be obtained by constructing a multidimensional ellipsoid containing all the scattered samples of the parameters while with a minimal volume (termed as "minimum volume method"), which, however, will bring about some severe difficulties. Firstly, optimization problems need to be formulated to determine the smallest ellipsoid, which can be effectively solved only for problems with very small numbers of parameters. With increasing of the parameters, the complexity of these optimization problems will grow remarkably as much more design variables and constraints will be involved. Especially for high-dimensional problems, it seems impossible to obtain a useful ellipsoid through the minimum volume method. Secondly, even though a precise ellipsoid can be created through this method, we can only have a rough knowledge on the whole uncertainty. Some important information such as dependence between the parameters and effects of the dependence on the systematic responses are still unavailable, which, however, is very important for our uncertainty analysis of complex engineering problems. Actually, the above problems have been the main obstacles influencing the practicability of the convex model approach.

This paper aims to create a mathematical foundation for correlation analysis of the non-probabilistic convex model, and whereby propose an approximate method to efficiently construct the multidimensional ellipsoids on the uncertainty. Through the present work, we expect to promote the engineering practicability of the convex model approach a certain extent. Four main parts are included in the following text. Firstly, a correlation analysis technique is proposed for convex model, based on which the multidimensional ellipsoids on the uncertainty can be created very easily and conveniently for problems with any dimensions. Secondly, by combining the correlation analysis technique with the non-probabilistic reliability index approach, a reliability analysis method is developed for uncertain structures. Thirdly, six numerical examples are provided to demonstrate the effectiveness of the present method. Finally, a conclusion is given.

2. Multidimensional ellipsoidal convex model

Assume that there exist n uncertain parameters X_i $i = 1, 2, \ldots, n$ describing either in the structural properties or in the excitations. These parameters constitute an n-dimensional parameter space, namely $\mathbf{X}^{\mathrm{T}} = \{X_1, X_2, \ldots, X_n\}$. Also assume that we have limited information on the uncertainty, represented by m experimental samples $\mathbf{X}^{(r)}$, $r = 1, 2, \ldots, m$ in this n-dimensional space. Then an ellipsoid containing all the samples can be created to quantify the uncertainty [28]

$$(\mathbf{X} - \mathbf{X}^0)^{\mathrm{T}}\mathbf{G}(\mathbf{X} - \mathbf{X}^0) \leqslant 1, \tag{1}$$

where $\mathbf{X}^0$ denotes the central point of the ellipsoid, and $\mathbf{G}$ is the characteristic matrix of the ellipsoid

$$\mathbf{G} = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ g_{n1} & g_{n2} & \ldots & g_{nn} \end{bmatrix}. \tag{2}$$

The characteristic matrix $\mathbf{G}$ determines the size and orientation of the ellipsoid, and it is symmetric positive-definite. Only when the axes of the ellipsoid are directed along the axes of the coordinates, $\mathbf{G}$ becomes a diagonal matrix. By using the convex model, all possible combinations of the uncertain parameters are assumed to fall into the above ellipsoid.

Actually, there exist an infinite number of ellipsoids containing the m samples, among which the one with a minimum volume is considered as the best one. Theoretically, such an ellipsoid can be obtained by solving a following optimization problem

$$\begin{aligned} &\min \prod_{i=1}^{n} r_i(g_{11}, g_{12}, \ldots, g_{nn}) \\ &\text{s.t. } (\mathbf{X}^{(i)} - \mathbf{X}^0)^{\mathrm{T}} \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ g_{n1} & g_{n2} & \ldots & g_{nn} \end{bmatrix} (\mathbf{X}^{(i)} - \mathbf{X}^0) \leqslant 1, \quad i = 1, 2, \ldots, n, \\ &r_i \geqslant 0, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{3}$$

where r_i, $i = 1, 2, \ldots, n$ denote the half lengths of all the axes of the multidimensional ellipsoid, and they can be obtained through an eigenvalue decomposition to the characteristic matrix $\mathbf{G}$. The optimization variables of the above problem are elements g_{ij}, $i = 1, 2, \ldots, n$, $j \geqslant i$ of the characteristic matrix $\mathbf{G}$ and the central point coordinates X_i^0, $i = 1, 2, \ldots, n$. The size of the ellipsoidal volume can be well quantified by the product of all the r_i, $i = 1, 2, \ldots, n$.

In theory the above minimum volume method may be an ideal way to create the ellipsoid on the uncertain parameters. Nevertheless, it should be pointed out that actually the above method is applicable only to a very small number of problems because of the following reasons. Firstly, an optimization problem in Eq. (3) needs to be solved when seeking the smallest ellipsoid, which will become extremely complex when the parameter dimension and the sample size become a little large, as in general dozens or even hundreds of constraints and optimization variables will be easily involved. Furthermore, in the optimization process all eigenvalues of the characteristic matrix $\mathbf{G}$ should be guaranteed to be positive in each iterate, otherwise, the obtained matrix may not correspond to an ellipsoid. Therefore, it seems not an easy job for the current optimization algorithms to always ensure a robust convergence for such a complex problem. Secondly, more importantly, the optimization problem in Eq. (3) in general has multiple optima, in which the global optimum is actually what we want, as only the global optimum can ensure a minimum volume of the ellipsoid. A local optimum for Eq. (13) may correspond to an ellipsoid which has a much larger volume than the real one, and generally we will obtain an ultra-conservative analysis or design based on such an overlarge ellipsoid. Nevertheless, for current optimization techniques the global optimality is still a theoretical and numerical difficulty. On the one hand, it is well