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Numerical method for the adhesive normal contact analysis based on a Dugdale approximation

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ABSTRACT

Modeling adhesion between two contacting surfaces plays a vital role in nano-tribology. However, providing analytical models, although desirable, is mostly impossible, in particular for complex geometries. Therefore, much attention has to be paid to numerical modeling of this phenomenon. Based on the adhesive stress description of the Maugis-Dugdale model of adhesion, which is credible over a broad range of engineering applications, an extended Conjugate Gradient Method (CGM) has been developed for adhesive contact problems. To examine the accuracy of the proposed method, the common case of the adhesive contact of a rigid sphere on an elastic half-space is investigated. To further evaluate the accuracy of this method, the adhesive contact of a rigid sphere over a wavy elastic half-space is also studied for different combinations of the amplitude and wavelength. There is good agreement between the analytical solution and the values predicted by the proposed method in the force-approach curves. Moreover, the calculation of pull-off force at a bisinusoidal interface between two surfaces is carried out for various cases to study the effects of different influential parameters including work of adhesion, elastic modulus, radius curvature at a crest, and the wavelength ratio. A curve is fitted on the calculated pull-off force in order to express it as an analytical relation. Similar to the JKR and DMT expressions for the pull-off force of a rigid ball on an elastic half-plane, the fitted curve is not affected by the elastic modulus and is linearly dependent on the radius of curvature and the work of adhesion. In addition, a power law governs the relation between pull-off force and the wavelength ratio. In the end, it is shown that roughness can either increase or decrease the adhesive force at a rough interface depending on the degree of the roughness.

1. Introduction

Adhesion plays a significant role in several technological fields and serves as one of the main reliability issues while dealing with smooth surfaces in contact under relatively low normal loads such as the case of micro/nano devices [1,2]. The early research on adhesion in contact mechanics was done by Bradley who studied the adhesive contact of rigid spheres [3]. Later on, two opposing classical theories of adhesion, JKR [4], and DMT [5], for single spherical elastic contacts were presented. Although these two models take different approaches and make significantly different assumptions, they are both true. It was shown by Tabor that these two models are the two opposite extreme limits of a single theory characterized by the Tabor parameter [6]:

$$\mu = \left(\frac{R\,\Delta\gamma^2}{E^{*2} z_0^3} \right)^{1/3} \tag{1}$$

where R, $\Delta\gamma$, E^*, z_0 are the radius of the sphere, work of adhesion, the effective elastic modulus, and the equilibrium separation, ranging from 0.2 nm to 0.4 nm. The JKR model is valid for large values of the Tabor parameter, as in the case of large and compliant contacts. The DMT model, however, is suitable for low values of this parameter, as for small and stiff contacts. Following these two models, Muller *et al.* developed a numerical solution to the adhesion interaction by integrating the Lennard-Jones potential and characterized the transition from DMT to JKR by adjusting the Tabor parameter [7]. Subsequently, Maugis provided a solution to this contact problem through assuming the contribution of adhesion inside and outside the contact area, by means of a Dugdale approximation and is known as Maugis-Dugdale

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(MD) model [8]. He defined an adhesive parameter which is equivalent to the Tabor parameter as:

$$\lambda = \sigma_0\left(\frac{9R}{2\pi\,\Delta\gamma E^{*2}}\right)^{1/3} = 1.16\mu, \qquad \sigma_0 = \frac{16\Delta\gamma}{9\sqrt{3}\,z_0} \tag{2}$$

in which σ_0 is the maximum attractive pressure of the Lennard-Jones potential. Based on the MD model, Johnson and Greenwood constructed an adhesion map for the contact of elastic spheres [9,10].

Although the mentioned analytical models provide exact solutions to the adhesion problem, they are limited to simple and smooth geometries. Thus, researchers have resorted to numerical approaches for surfaces with a more complex geometry [11–13]. Several authors have attempted to numerically evaluate the adhesion between two rough surfaces through multi-asperity and finite element approaches. In multi-asperity models, the surface is described merely in terms of the summit geometry and the rest of the surface is discarded [14–16]. As the main limitation of this model, next to the simplified summit geometry, is the assumption of a Gaussian distribution of roughness height, which is not valid for many engineering applications, different height distributions have been implemented, all of which still have the limitation to a specific application [16–19]. Finite element models for adhesive contact problems, incorporating the Lennard-Jones potential into the framework of nonlinear continuum mechanics, have also been developed [20,21].

The roughness of a surface could be described by means of surface models, such as fractals and Fourier transforms [22–25]. In these cases, numerical simulation of an adhesive contact has been considered while taking into account the regenerated topography of the contacting surfaces and not the original topography as it is measured. Here, the measured topography of a surface can be different from the roughness details regenerated or approximated by stochastic parameters. Consequently, since the adhesion force is a function of the exact local distance between the asperities of the two contacting surfaces, changing this distance influences the corresponding local adhesive force, and thus, deviation in the adhesive behavior is expected. Therefore, the core purpose of the current study is to develop a numerical adhesive contact solver between two elastic surfaces without any assumption on or restriction to the topography of the surfaces. Restricting ourselves to this goal, the Conjugate Gradient Method (CGM) is considered. CGM is a fast and accurate numerical algorithm typically implemented for a system of linear equations and is often used in an iterative scheme [26]. Polonsky and Keer first implemented this method for non-adhesive normal contact problems [27]. Ever since, this method has been extensively exploited for various non-adhesive contact problems in order to determine the normal and tangential contact stresses and contact area [28–33].

In the present study, the CGM is extended to include a Dugdale approximation, similar to MD model of adhesion, for the adhesive stress. In this way, it is used for the adhesive contact analysis between two elastic bodies with a general complex surface geometry.

2. Adhesive parameters

Maugis represented the surface force in terms of a Dugdale cohesive zone and stated that adhesion is present up to a specific value of the separation between the two contacting bodies, named h_0. Within this separation, the attractive pressure of σ_0, is applied such that [8] (Fig. 1):

$$\Delta\gamma = \sigma_0\, h_0 \tag{3}$$

This results in $h_0 = 9\sqrt{3}z_0/16 = 0.974z_0$. Based on the definition of the MD model, the pressure inside the contact region is the superposition of the positive Hertzian pressure of radius a and the negative adhesive pressure. Outside the contact region, the attractive pressure is constant over a ring of inner and outer radii of a and c, in which:

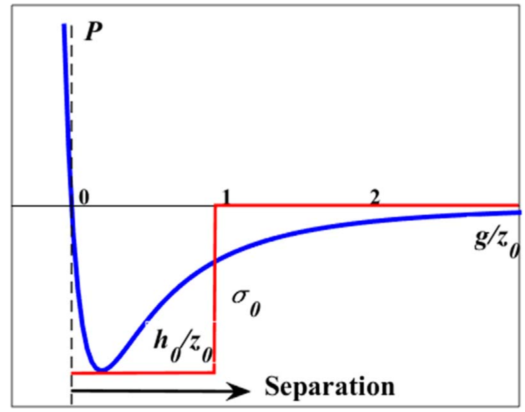

Fig. 1. Dugdale approximation (red line) of Lennard-Jones potential (blue line). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article).

$$\varphi = \begin{cases} 0 & at \quad r = a \\ h_0 & at \quad r = c \end{cases} \tag{4}$$

where φ is the separation. The Dugdale stress, $-\sigma_0$, and the maximum separation, h_0, are the two adhesive parameters that will be used in the proposed algorithm for the adhesive normal contact between two bodies.

3. Problem definition

When two rough surfaces are brought into contact, the generated normal stress (pressure) deforms the surfaces. The composite deformation of the two surfaces, $u(x, y)$ due to the applied pressure,$P(x, y)$ over the region Ω is given by:

$$u(x, y) = \int_\Omega k(x-\zeta, y-\eta)P(\zeta, \eta)d\zeta d\eta \tag{5}$$

where x and y are the spatial coordinates and $k(x, y)$ is the Boussinesq kernel function and is expressed as [34]:

$$k(x, y) = \frac{1}{\pi E^*}\frac{1}{\sqrt{x^2+y^2}}, \qquad \frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \tag{6}$$

in which E_i, ν_i, $i = 1, 2$ are the elastic moduli and Poisson ratios of the two contacting surfaces. If the separation between these two surfaces before and after the deformation are denoted by $h(x, y)$ and $g(x, y)$, they can be related to the deformation $u(x, y)$ as:

$$g(x, y) = u(x, y) + h(x, y) - \delta \tag{7}$$

where δ is the rigid approach of the two surfaces (Fig. 2). The non-adhesive contact problem necessitates the pressure to be positive at contacting areas, where there is no separation between the two surfaces (where $g(x, y) = 0$). On the other hand, at separate areas (where $g(x, y) > 0$), the pressure must be zero. Moreover, the pressure distribution must balance the applied normal load, F_0. In other words:

$$\begin{aligned} &P(x, y) > 0 \quad at \quad g(x, y) = 0 \\ &P(x, y) = 0 \quad at \quad g(x, y) > 0 \\ &\int_\Omega P(x, y)\, dx\, dy = F_0 \end{aligned} \tag{8}$$

The adhesive contact problem is, nevertheless, different from the definition by Eq. (8). For an adhesive contact problem, there is a negative stress between separated areas described by the Lennard-Jones potential as an explicit function of the local separation. As stated in the previous section, the MD model of adhesion assumes this dependence to be a step function of the local separation (by means of a Dugdale approximation of the Lennard-Jones expression). Based on this description, the negative stress due to adhesion at separated areas