Tribology International 114 (2017) 358–364

Contents lists available at ScienceDirect

Tribology International

journal homepage: www.elsevier.com/locate/triboint

Surface normal deformation in elastic quarter-space

W. Wang[a], L. Guo[b], P.L. Wong[b,*], Z.M. Zhang[a]

[a] School of Mechatronics Engineering and Automation, Shanghai University, Shanghai, China
[b] Department of Mechanical and Biomedical Engineering, City University of Hong Kong, Hong Kong, China

ARTICLE INFO

Keywords:
Elastic quarter space
Surface deformation
Flexibility matrix

ABSTRACT

An efficient and explicit solution for the surface deformation of quarter-space under normal load is developed using the concept of flexibility matrix, which serves like springs in response to loads. Quarter-space is characterized by the unbounded side surface, such as in roller bearings and gears. The solution method is verified using a typical case. The edge effect on surface deformation under three load types namely, Hertzian point, flat cylindrical punch and Hertzian line, are evaluated. The effect can be considerable if the applied load is close to edge. The flexibility matrix is constant for a given case. Hence, the solution method is highly efficient, and particularly suitable for quarter-space problems which require iterative calculations, such as elasto-hydrodynamic lubrication analyses.

1. Introduction

Acquiring the elastic deformation of contact surfaces is important in engineering. The solution process also needs to be fast and efficient for certain applications that require iterative calculations for the surface deformation, such as the analysis of tribo-pairs operating under the elasto-hydrodynamic lubrication (EHL) regime. Some common engineering components, such as roller bearings, gears and cam-followers, are characterized by the existence of free edge surfaces. Contact problems of these components are, in fact, more accurately modeled by elastic quarter-space (Fig. 1(a)). Nevertheless, the available solutions of elastic quarter-space are very complex, such that the elastic half-space model (semi-infinite body) is widely adopted for calculating contact stress and deformation in practical mechanical systems, such as those aforementioned applications, for their contact solutions are readily obtained with Bussinessq or Love formulae [1,2]. The assumption of semi-infinite body model is obviously not satisfied in these practical cases. For example, in the contact of gears and roller bearings, the length of the gear tooth and bearing roller are finite. Thus, the effect of free edge surfaces cannot be ignored and these components cannot be taken as semi-infinite bodies. The elastic quarter-space model is, indeed, more appropriate.

Hetenyi [3] tackled the quarter-space problem with the concept of iteratively overlapping mutually orthogonal half-spaces with mirrored load pairs till fulfilling the boundary conditions of the quarter-space. Keer et al. [4] utilized Hetenyi's overlapping half-space idea and derived two integral equations to describe the quarter-space problem. They solved the equations with Fourier transform. Nevertheless, their method can only be applied to cases where the load can be Fourier-transformed. Later on, Hanson and Keer [5] overcame this limitation with a direct numerical solution of the quarter-space by solving two dimensional integral equations. Thus, any load type can be considered. The difference in the stress obtained with quarter-space and half-space models was studied. As pointed out in [5], the magnitude and position of the maximum stress calculated with quarter-space and half-space vary, especially when the load is located in the immediate neighborhood of the free end. Guilbault [6] made use of a correction factor which multiplies the Hetenyi's mirrored loads to simultaneously correct the influence of stresses on elastic deformation of a quarter-space, which provides a much faster solution than a complete Hetenyi process. This correction factor method was applied by Najjari and Guilbault [7] to investigate the edge effect in EHL analysis of roller bearings. Nevertheless, this method gives only approximate solutions. The present authors [8] have recently obtained the limit of Hetenyi's iteration with a matrix method and developed an explicit solution to the stress field of quarter-space problems. The aforementioned methods are all based on the overlapping half-space concept of Hetenyi. Apart from these, Bower et al. [9] adopted finite element method (FEM) to analyze the ratcheting limit of rail's plastic deformation. Hecker and Romanov [10] applied Mellin transform to solve the stress distribution of quarter-space. Ritz's method was also applied by Guenfoud et al. [11] to obtain the displacement solution of quarter-space.

There is another perspective of the contact problem of quarter-space by considering its contact with a rigid body. Gerber [12] was the first to study a contact between a rigid body and an elastic quarter-

* Corresponding author.
E-mail address: meplwong@cityu.edu.hk (P.L. Wong).

http://dx.doi.org/10.1016/j.triboint.2017.04.044
Received 23 February 2017; Received in revised form 20 April 2017; Accepted 25 April 2017
Available online 29 April 2017

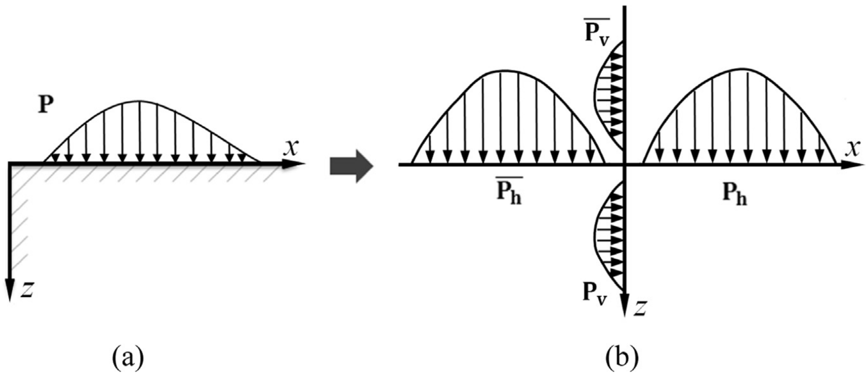

Fig. 1. Quarter-space solutions equivalent to overlapping half-spaces.

space. He obtained the stress distribution in a quarter-space which is pressed by a rectangular punch. Keer et al. [13] studied a quarter-space loaded with a rigid cylindrical indenter by integral transform techniques. Hanson and Keer [14] solved the contact stresses between a spherical indenter and a quarter-space. Wang et al. [15] studied the problem of a quarter-space in contact with a rigid sphere using equivalent inclusion method. Zhang et al. [16,17] analyzed the contact of a rigid roller and a finite-length elastic body. To include the effect of the free edge surfaces of the elastic body, they applied the method of Zhang et al. [8] in the study. However, the shear stresses on a free end surface as induced by the mirrored loads on the plane of the other end surface cannot be eliminated, i.e. it does not fulfill the zero stress boundary condition of the free surface.

The elastic deformation of quarter-spaces is needed in the solution of many engineering problems, such as rail/wheel contacts [14] and EHL analyses of roller contacts [18–20]. The analyses of these application examples require iterative calculations. Thus, it requires not only accurate but also efficient solution for surface deformation of elastic quarter-space. In this paper, the solution of surface deformation of a quarter-space is developed resembling a matrix of springs. The deformation of a spring is obtained by simply dividing the load over its stiffness, or multiplying the load with its flexibility. If such a simple process can be implemented into the calculation loops of the above examples, the solution process would be significantly simplified and shortened. The complete calculation times can thus be much lower, especially if a great many times of iteration is needed. In order to realize this, a characteristic property of elastic quarter-space concerning its stiffness or flexibility must be known prior to entering the loop of calculation. This characteristic property must be independent of the load, and derived without the knowledge of the current load. The present paper achieves this aim by extending our recently proposed technique [8] for quarter-space solutions, such that an explicit form of the flexibility matrix of the elastic quarter-space is derived. This flexibility matrix is independent of the load, so that it can be used in every loop of the iteration process. The elastic surface deformation can be immediately obtained by simply multiplying this matrix with the current load distribution. The theoretical derivation of the flexibility matrix for the elastic surface deformation with the quarter-space model is presented. The solution method is also validated through a special case study. The difference in the surface normal deformation calculated with quarter-space and half-space models are not yet investigated comprehensively. Therefore, the results of the surface normal deformation of quarter- and half-space models under different typical loads are also presented and discussed.

2. Solution of surface normal deformation with quarter-space model

2.1. Derivation of flexibility matrix

A quarter-space problem with a distribution load **P** on the top surface as shown in Fig. 1(a) can be solved by making use of the solutions of two mutually orthogonal half-spaces as shown in Fig. 1(b), which is based on the overlapping half-space idea of Hetenyi [3]. To solve a quarter-space problem with matrix formulation [8], the horizontal and vertical surfaces of the quarter-space are discretized with rectangular meshes of different sizes. Fig. 2(a) shows schematically the mesh pattern on the top surface. The region near the free edge surface is discretized with finer meshes in order to enhance the accuracy of the deformation results close to the free edge. The solution of Fig. 1(a) is obtained by superimposing the half-space solutions of load-pairs: $\mathbf{P_h}, \overline{\mathbf{P_h}}$ and $\mathbf{P_v}$, $\overline{\mathbf{P_v}}$ ($\overline{\mathbf{P_h}}$ and $\overline{\mathbf{P_v}}$ are mirror loads of $\mathbf{P_h}$ and $\mathbf{P_v}$, respectively). Making use the stress boundary conditions of the original quarter-space (Fig. 1(a)): **P** on the top surface and zero stress on the vertical side surface, explicit solutions of the equivalent load $\mathbf{P_h}$ and $\mathbf{P_v}$ of the half-spaces (Fig. 1(b)) can be readily obtained by [8],

$$\mathbf{P_h} = \mathbf{A} \bullet \mathbf{P} \tag{1}$$

$$\mathbf{P_v} = \mathbf{B} \bullet \mathbf{P} \tag{2}$$

The Appendix shows how the two coefficient matrices **A** and **B** can be calculated. All pressure distributions are in matrix format as,

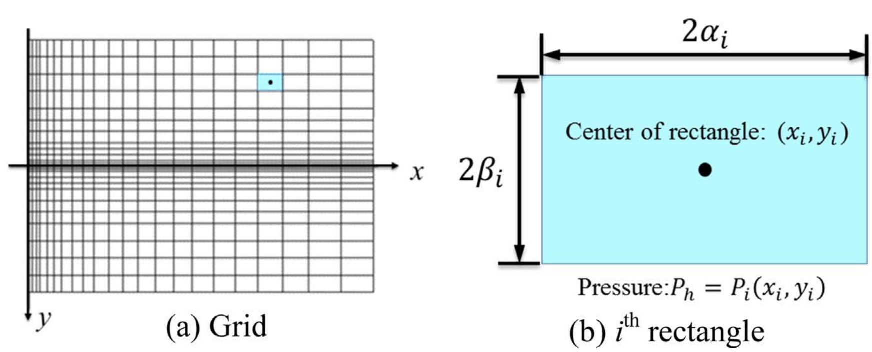

Fig. 2. (a) Mesh pattern and (b) the i^{th} patch on top surface.