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On the use of DMT approximations in adhesive contacts, with remarks on random rough contacts

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ABSTRACT

The contact between rough surfaces with adhesion is an extremely difficult problem, and the approximation of the DMT theory (to neglect deformations due to attractive forces), originally developed for spherical contact of very small radius, is receiving some new interest. The DMT approximation leads to extremely large overestimations of the adhesive forces in the case of spherical contact, except at pull-off. For cylindrical contact, the opposite trend is found for larger contact areas. These findings suggest some caution in solving rough contacts with DMT models, unless the Tabor parameter is really low. Further approximate models like that of Pastewka & Robbins' may be explained to work only due to a coincidence of error cancellation in their range of parameters.

1. Introduction

The Derjaguin-Muller-Toporov (DMT) theory [5,10,11], for the contact of elastic spheres with adhesion, has a long history. After Bradley [1] and Derjaguin [4] obtained the adhesive force between two *rigid* spheres, equal to $2\pi Rw$, where w is the work of adhesion, and R is the radius of the sphere, JKR [13] developed a theory for elastic spheres, assuming adhesive forces occur entirely within the contact area, obtaining 3/4 of the Bradley pull-off value, and hence the independence on the elastic modulus raised a long debate about the a comparison of the pull-off prefactor.

As the main attention in the diatribe between JKR and DMT was limited to the pull-off value, it is often believed that DMT is the limit for Tabor parameter [20].

$$\mu = \left(\frac{Rw^2}{E^{*2}\Delta r^3}\right)^{1/3} = \frac{(Rl_a^2)^{1/3}}{\Delta r} = \frac{\sigma_{th}}{E^*}\left(\frac{R}{l_a}\right)^{1/3} \to 0 \tag{1}$$

where Δr is the range of attraction of adhesive forces, close to atomic distance, and E^* the plane strain elastic modulus. Also, we have introduced the length $l_a = w/E^*$ as an alternative measure of adhesion, and σ_{th} is the theoretical strength of the material. Now, while it is true that DMT predicts the Bradley result for the force at pull-off also for elastic spheres, the DMT theories have been much less compared with exact results, when considering the entire load-displacement curves. In both DMT methods,

- the attraction forces act exclusively *outside* the contact, and
- the repulsive forces *only* are responsible for deformation.

Then, in the DMT "force method"

- the force of adhesion can be simply obtained by integrating, according to Derjaguin's approximation, the forces of facing elements outside of the contact, separated by a gap which is given by Hertz theory.

We shall concentrate on the latter (force) method, which is what is commonly used when DMT approximation is considered in the generalized context of rough contact (see [19]). In one looks at the force of adhesion *not at pull-off,* it decreases from $2\pi Rw$ to πRw, in the "thermodynamic method"[1] while it increases in the "force method", as shown by [11], and Pashley [12]. Pashley [12] in particular notices that in the force method, the adhesive force should be always larger than $2\pi Rw$, the value obtained for a truncated rigid sphere independently on the contact radius, as the Hertzian profile is closer to the flat surface than the rigid spherical profile.

Maugis [14], in his Maugis-Dugdale analysis (which *does not* make the DMT approximations) called the low μ end the "DMT theory", which in fact is now the version most commonly associated with DMT, and sometimes called DMT-M. In this version, the attractive forces are constant, and equal to the pull-off value, $2\pi Rw$. This is indeed what comes out from DMT theory, but only in the limit of $\mu = 0$: therefore,

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[1] In the "thermodynamic method", the force is computed by the rate of change of surface energy as the sphere is pressed with approach α, i.e. $dW_s/d\alpha$. It turns out that the "thermodynamic" method tends to give opposite error with respect to the "force method", and it is also more complicated to use, so it has not received much attention.

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DMT is exact only in this limit case, and for any finite value of μ, DMT theories give an error which we shall estimate in fact in details in the present paper as a function of the Tabor parameter, since the previous estimates of [11], and Pashley [12] do not clarify clearly the role of Tabor parameter. Greenwood [7] also has discussed more details of the DMT theory in the limit $\mu \to 0$.

But we shall not limit ourselves to the spherical contact case, since this case has been given already much attention, and is only one special case. The DMT approximation is gaining relevance more recently again, in the context of rough contact, where there is a lot of interest in simplifying the problem since the JKR assumption leads to very complicated and hysteretic behaviour, which so far, has not been included in a framework of any theory, despite some attempts [2,18]. Moreover, as roughness at the small scales seems to point to low values of Tabor parameter, the "almost rigid" behaviour has some fundamental interest. Persson & Scaraggi [19] have indeed attempted using the DMT approximations using the Persson's theory for adhesionless contact, and seemed to find some reasonable accuracy at least for the range of parameters they observed. Also, Pastewka & Robbins [17] PR in the following make some scaling predictions which seem to fit well some limited range of their extensive full numerical simulations involving atomistics rough solids. We made a first attempt to discuss PR findings in Ciavarella [3] where we noticed that, if PR were concerned with spherical contact, using the DMT approximation with the additional simplification of using only the asymptotic first term in the expansion of the gap outside the Hertzian contacts, they would find easily large errors. But one limit to this estimate is that we assumed circular contact, whereas PR calculation shows more like 2D fractal contact area, perhaps closer to very elongated contacts like in 2D cylindrical contact—indeed, as we will discuss below, they find a characteristic diameter of the contact independent on load, and load only affects the elongation of the contact area. Therefore, in the present note we develop a simple 2D line contact DMT model, we give more details about the DMT limit for the sphere, and make further comparisons with the DMT rough contact results.

2. A 2D DMT-Maugis line contact model

For 2D contact with "repulsive" diameter $d_{rep} = 2a$, the full form of the gap outside the contact is

$$\frac{h(c)}{a} = \frac{a}{R} f\left(\frac{c}{a}\right) \tag{2}$$

where $c > a$ and [9].

$$f\left(\frac{c}{a}\right) = \frac{1}{2}\left[\frac{c}{a}\sqrt{\left(\frac{c}{a}\right)^2 - 1} - \log\left[\frac{c}{a} - \sqrt{\left(\frac{c}{a}\right)^2 - 1}\right]\right] \tag{3}$$

whose first term in the series expansion near c=a is $f_{as}\left(\frac{c}{a}\right) = \frac{\sqrt{8}}{3}\frac{a}{R}\left(\frac{c}{a} - 1\right)^{3/2}$, is used in the PR version of DMT method, as commented in [3], and in the later paragraph. We shall assume for the potential, a Maugis simple law. This will permit a direct comparison with the "exact" Maugis solution including deformations induced by the adhesive stresses, given by Baney and Hui (1997) Morrow and Lovell [15] and Johnson and Greenwood [9], whereas Jin et al. [8] give a double Hertz solution which show that results will not differ much with those with other choices of potential.

The pull-off force is not a simple multiple of Rw as with circular contacts, but varies from $P_{rigid} = \sqrt{8R\sigma_{th}w}$ to $P_{JKR} = \frac{3}{4}(4\pi E^* R w^2)^{1/3} = \frac{3}{4^{2/3}}(\pi E^* R w^2)^{1/3}$, so it depends on elastic modulus.

We define the following non-dimensional contact radius and load

$$a^* = \frac{a}{2\pi^{1/3}R^{2/3}l_a^{1/3}} \tag{4}$$

$$P^* = \frac{P}{(\pi E^* R w^2)^{1/3}} = \frac{P}{4^{2/3}P_{JKR}/3} \tag{5}$$

and accordingly the Hertz and JKR limits are found as [9].

$$P^*_{Hertz} = a^{*2} \tag{6}$$

$$P^*_{JKR} = a^{*2} - 2\sqrt{a^*} = P^*_{Hertz} - 2\sqrt{a^*} \tag{7}$$

whereas the Maugis-Dugdale model shows a smooth transition between the Hertz and JKR limit — unlike the 3D case, where there is a transition from Bradley rigid to JKR model. Notice that the rigid limit is subtle: while there is a tendency to the Hertz regime, the actual pull-off in rigid limit is not zero.

Moving to a DMT force method estimate, setting the gap to Δr gives

$$f\left(\frac{c}{a}\right) = \frac{R\Delta r}{a^2} \tag{8}$$

and it is clear that the approximation is good until $\frac{R\Delta r}{a^2} < 1$. In dimensionless notation, $R\Delta r/a^2 = (4\pi^{2/3}a^{*2}\mu)^{-1}$.

Using the asymptotic term, the lateral distance defining the size of attractive region (which is composed of two strips of size d_{att}) is

$$d_{att,asym} = \left[\frac{\left(\frac{3}{2}R\Delta r\right)^2}{d_{rep}}\right]^{1/3} \tag{9}$$

and when contact radius is small, we require a correction from the solution of (8), $\frac{d_{att}}{a} = \frac{c}{a} - 1 = \beta\frac{d_{att,asym}}{a}$. As we are using the Maugis potential, the attractive load is therefore simply the product of the theoretical strength and the area of the adhesive strips, $P_{DMT,att} = 2d_{att}\frac{w}{\Delta r}$. Using (9), the attractive load is obtained as

$$P^3_{DMT,att} = -2^3\beta^3 d^3_{att}\left(\frac{w}{\Delta r}\right)^3 = -2^3\beta^3\left(\frac{3}{2}\right)^2 R^2\Delta r^2\frac{1}{2a}\left(\frac{w}{\Delta r}\right)^3 \tag{10}$$

Using ((4), (5)) and Tabor parameter (1),

$$P^*_{DMT,att} \simeq -\beta\left(\frac{\mu}{a^*}\right)^{1/3} \tag{11}$$

The results of the DMT theory are presented in Fig. 1a (dashed lines) for $\mu = 0.05, 0.25, 1, 5$, together with the Maugis solution of Johnson and Greenwood [9] which we take as reference as "exact". In Fig. 1b, we compare the JG Maugis solution with the further approximation of taking only the first term in the gap profile, which clearly leads to serious errors even at low Tabor parameter. We shall explain why in the PR use of this further approximation, the error was probably balanced by another approximation.

It is clear that the DMT theory gives a reasonable result only for $\mu < 0.05$ as at $\mu = 0.25$, the error is already significant, of the order of 20% at pull-off. Errors become large at $\mu > 1$, particularly at pull-off, as larger than 100%. This is clearer from Fig. 2, where the pull-off values are plotted.

3. The spherical DMT model

As this case is classical, DMT has been compared with JKR and other models in a large number of papers. However, the comparison is mostly done for the value at pull-off, where of course the DMT model gives the correct Bradley result for $\mu = 0$: only perhaps [11], Pashley [12], and Greenwood [7] discuss more details of DMT at compressions larger than zero, and even they, do not fully clarify the error as a function of Tabor parameter.

We shall assume for the potential, a Maugis simple law, consistently to the line contact of the previous paragraph. We have already discussed a further simplified form of this model in [3] inspired by PR paper [17], namely using the first term asymptotic form of the gap function outside of the contact, and of computing the area of attraction