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Combined optimization of bi-material structural layout and voltage distribution for in-plane piezoelectric actuation

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ABSTRACT

This paper investigates the combined optimization of bi-material structural layout and actuation voltage distribution of structures with embedded in-plane piezoelectric actuators. The maximization of the nodal displacement at a selected output port is considered as the design objective. A two-phase material model with power-law penalization is employed in the topology optimization of the actuator elements and the coupled surrounding structure. In order to incorporate the actuation voltage directly into the design for achieving the best overall actuation performance, element-wise voltage design variables are also included in the optimization. For the purpose of easy implementation of the electric system, the allowable voltage levels at an individual element are confined to three discrete values, namely zero and two prescribed values with opposite signs. To this end, a special interpolation scheme between the tri-level voltage values and the design variables is used in the optimization model. Based on the design sensitivity analysis of the objective function, the combined optimization problem is solved with the MMA algorithm. Numerical examples are presented to demonstrate the applicability of the proposed optimization model and numerical techniques. The optimal solutions also confirmed that larger output displacement can be achieved by introducing voltage design variables into the design problem.

1. Introduction

An appealing characteristic of piezoelectric materials is that they produce a mechanical strain when an electrical field is applied and vice versa. Piezoceramics can deliver up to 0.4% strain. Recently, piezoelectric single-crystals that can produce greater than 1% strains are already available. Actuators based on piezoelectric effects are attractive candidates for scenarios requiring compact size, high energy density, high spatial resolution and fast time response. Therefore, piezoelectric actuator-integrated structures are suitable for a wide range of applications such as high precision actuation, microvalve/micropump, aerodynamic flow control and active shape control. Laminated plates with imbedded piezoelectric bimorph/unimorph actuators are most commonly used architectures for generating off-plane motions. However, innovative designs of flexible structures or compliant mechanisms incorporating in-plane piezoelectric actuation are also highly desirable in many potential applications, such as biomedical devices and MEMS.

Many studies have been devoted to the design optimization of active structures incorporating piezoelectric actuators. Conventionally, the optimal synthesis of piezoelectric-actuation-based active structures assumed that the topologies of actuator elements are predetermined [1], which imposes an undesired restriction to the design problem. Recently, optimal placement of conventional materials or piezoelectric materials by topology optimization methods has drawn considerable attentions. Topology optimization aims at the optimal redistribution of a given amount of material in the design domain for achieving the best structural performance like stiffness, flexibility and natural frequencies [2,3]. Various methods are now available for topology optimization and the homogenization method [4], the SIMP (Solid Isotropic Material with Penalization) method [5] and the level set method [6,7] have become popular in academic research and practical applications. The topology optimization of active structures utilizing piezoelectric effects has been studied by a number of authors. Maddisetty and Frecker [8] presented the topology synthesis of a piezoceramic actuator together with a compliant mechanism by selectively removing certain passive elements or active elements from a predefined planar truss structure according to sizing optimization results. Under the condition of given shapes and positions of piezo-actuators, the optimal placement of conventional materials in electromechanical devices was studied by Silva [9], Silva et al. [10] and Carbonari et al. [11]. Based on a ground structure approach, Mukherjee and Joshi [12] and Sun and Tong [13] presented heuristic

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iterative procedures for design optimization of piezoelectric actuator patterns in static shape control of plates. Kögl and Silva [14] extended the material density-based approach to the optimal distribution of piezoelectric materials in the actuator design for maximizing specified output displacements of plates under given electric fields. Zhang et al. [15] introduced electrode density design variables into the topology design of in-plane actuators made of piezoelectric materials. In the obtained optimal results, the areas with non-zero electrode densities are interpreted as the actuation parts while those with zero electrode density are interpreted as supporting parts. A few attempts have also been made to simultaneously optimize the placement of both conventional and piezoelectric materials. In a study reported by Buehler et al. [16], a homogenization topology optimization method is employed for distributing the conventional and the smart materials in a cantilever beam. Carbonari et al. [17] employed the bi-material SIMP approach in solving a similar design problem. In a study by Carbonari et al. [18], topology optimization method is applied to the design of piezoelectric micro-tools. Therein, the structural topology and the piezoceramic property gradation are optimized simultaneously. Recently, topology optimization of energy-harvesting devices based on piezoelectric effects has also drawn much attention [19–21].

It is noted that in the abovementioned formulations aiming for acquiring the best actuation performance, the material distribution is optimized under a specified (usually spatially uniform) electrical field or electric charge. A methodology that integrates layer-wise topology synthesis and actuation voltage optimization was developed by Kang and Tong [22] for laminated piezoelectric plates used in the shape control applications. Kang and Tong [23] also extended the penalization technique used in the SIMP approach to the single channel actuation voltage optimization by introducing a new interpolation function for relaxation of the tri-level optimization problem. Their studies showed that the introduction of the actuation voltage variables into the design optimization may -significantly increase the performance and functionality of the laminated smart plates.

One of the essential advantages of piezoelectric in-plane actuation over some of its counterparts like thermomechanical actuation is that a more spatially complex strain field can be induced at high frequency band-width by applying an electric field with well-designed spatial distribution. To make use of this benefit and achieve the best overall system design, one needs to fully exploit the interaction between the actuator elements and the passive components, as well as the coupling between the actuator elements and the actuation voltage. This therefore raises the need of combined design optimization of the structural topology together with the actuator positions and the actuation voltage. Though simultaneous topology optimization of the actuators and their coupling with surrounding structures has been addressed recently as mentioned above, more rewarding results are still expected by directly including the applied voltage into the design optimization.

In this paper, a mathematical formulation as well as associated numerical techniques for the rational design of active flexible structures with embedded in-plane piezoelectric actuators are presented. It is intended to provide a designer with a numerical tool of developing a meaningful conceptual design in the early stage of the design process. In the optimization problem, the layouts of the piezoelectric actuators and the coupling flexible structures, as well as the actuation voltage, are to be simultaneously optimized for achieving the best performance of a planar active structure. The considered structures are planar sheets that have two layers symmetrically bonded to a relatively thin metal shim. The two layers may consist of void, conventional material as passive parts (supporting structure) and piezoelectric material serving as in-plane actuators, as schematically illustrated in Fig. 1. The actuation regions are covered by electrodes on both outer surfaces while the metal shim is connected to electric ground and thus has a zero electrical potential. The actuation voltage is applied across the thickness of the piezoelectric layers, as shown in Fig. 2. The piezoelectric material in both actuation layers has the same poling direction; therefore the actuator delivers only in-plane deformations under applied actuation voltage. In order to accommodate an easily implemented electric equipment and to avoid impractically complex electrode configuration, we limit the actuation voltage output to V and $-V$ (see Fig. 2), where V is a prescribed value.

The remainder of this paper is organized as follows. In Section 2, the finite element modelling of the considered piezoelectric structures is briefly introduced. Section 3 describes the formulation of the combined optimization problem. In the optimal design problem, the SIMP approach based on the bi-material interpolation model proposed by Sigmund [24] is employed for the optimal placement of conventional and piezoelectric material. At the same time, the tri-level actuation voltage optimization technique suggested by Kang and Tong [23] is incorporated into the optimization model for determining the direction of the electrical fields to be applied across the plate thickness. In the subsequent section, the design sensitivity analysis is presented. Numerical examples will be given in Section 5 for illustrating the validity and the applicability of the proposed method. Finally, conclusions and further discussion are presented.

2. Finite element analysis

The constitutive model that describes the converse and the direct piezoelectric effects of a linear piezoelectric material is expressed as

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \\ D_i &= e_{ikl}\varepsilon_{kl} + \kappa_{ik}E_k, \end{aligned} \tag{1}$$

where σ_{ij}, C_{ijkl} and ε_{kl} are the mechanical stress tensor, the elasticity tensor, and the mechanical strain tensor, respectively; e_{ijk}, E_k, D_i and κ_{ik} are the piezoelectric stress tensor, the electric field vector, the electric displacement vector and the permittivity tensor, respectively. The electric field E_k is related to the electric potential φ by $E_k = -\varphi_{,k}$, where the subscript $(\cdot)_{,k}$ denotes the spatial derivative in the kth direction.

The principle of virtual work of a body-force free piezoelectric structure is given by

$$\int_\Omega (\sigma_{ij}\delta\varepsilon_{ij} - D_i\delta E_i)\mathrm{d}\Omega - \int_S (\bar{t}_i\delta u_i + \bar{q}\delta\phi)\mathrm{dS} = 0, \tag{2}$$

where $\bar{t}_i$ represents the surface tractions and $\bar{q}$ the surface charge density. From discretization of the integrals in Eq. (2) by the routine

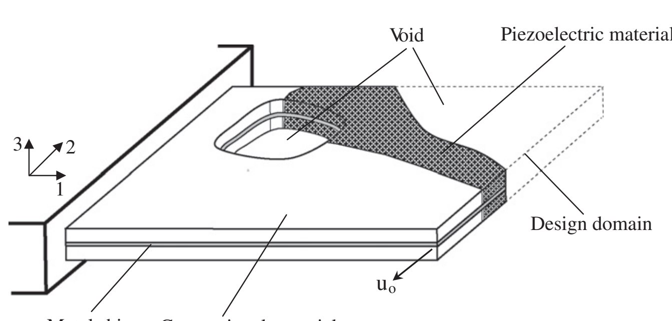

Fig. 1. Layer configuration of the three-layered active planar structure.