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Miscalculation of film thickness, friction and contact efficiency by ignoring tangential tractions in elastohydrodynamic contacts

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A B S T R A C T

Traction in elastohydrodynamic contacts causes normal surface displacements, affecting the lubricant film thickness. This is ignored in the vast majority of published theoretical studies and film-thickness regression formulae. This article analytically formulates traction effects on normal displacements, establishing the most contributing factors and the associated displacement error when tractions are ignored, which can exceed 20%. Numerical tests on a rolling-sliding-spinning elastohydrodynamic elliptical contact and comparison with well-known published formulae show that the central film thickness is unaffected by the traction factor simplification but the minimum film thickness is occasionally significantly overestimated. Traction and contact efficiency are also in error but usually by less than 1%. Correspondingly, wear may also be miscalculated, particularly in contacts with sliding and/or spinning motion.

1. Introduction

Adopting a scientific theory presupposes that the theory has been validated or has passed the test of time. Even so, there are cases where errors in mathematical modelling can go unnoticed for reasons such as: (a) when theoretical and experimental results agree in a given set of operating conditions because that set excludes conditions where a disagreement would have surfaced; (b) when normalized differences between theoretical and experimental results fall below an agreed error tolerance; (c) when in a set of evaluation parameters, there is good agreement between theoretical and experimental results for some parameters that are deemed important and poor agreement for others that are somehow deemed unimportant by the evaluator. Moreover, adopting a scientific theory should be done in conformance to its known limitations and in the same context the theory was originally constructed.

Even when a modelling error is finally exposed, it is not certain that future studies will account for it. An example concerns the inconsistency in applying the Reynolds equation for elastohydrodynamic lubrication (EHL) [1]. The original equation popularized in the 1950s is based on assuming constant fluid viscosity. However, subsequent application of the original equation on EHL problems by everyone since then allowed the dependence of viscosity on pressure as an after-treatment. That is, without re-deriving a Reynolds-type equation from the Navier-Stokes equations that would correctly address the pressure dependence of viscosity with additional partial derivatives in the equation. Although the correct treatment of the problem and the associated numerical errors have been detailed by Rajagopal and Szeri [1] since 2003, researchers still use the inconsistent *a posteriori* treatment either as a compromise in numerical precision, or because they have invested a lot of resources in developing and validating complex and not easily manageable computational codes in the past, or, simply, out of ignorance.

A similar modelling inefficiency is exposed in the present study. It also concerns the classic equations used to model EHL starting with the eminent work of Dowson and Higginson [2]. Specifically, in calculating the film thickness separating two counter-surfaces in an elastohydrodynamic (EHD) contact, it is necessary to calculate the normal elastic displacements in the contact, which form part of the film thickness. This is accomplished by using equations of the theory of elasticity in the context of contact mechanics [3]. Naturally, when two surfaces are in contact and in relative motion, even in the presence of a separating liquid film, they sustain both pressure and friction. Pressure causes normal elastic displacements of the EHL counter-surfaces and is accounted for in every published study to date. However, friction or traction in the tangent plane of the contact (tangential traction) also causes normal displacements, affecting both the film thickness and pressure distribution, yet this effect is ignored in the literature. In fact, there is no hint about it even in popular textbooks on EHL such as those by Dowson and Higginson [2] (chapter 5), Gohar [4] (section 4.7) and Hamrock et al. [5] (section 18.3 for rectangular contacts and 19.2 for elliptical contacts) to name but a few examples, although it is well documented in contact mechanics books such as Johnson's classic textbook [3] (e.g., in its section 2.5(b)) in regards to dry contacts. This means that in solving the Reynolds equation, an EHD contact is effectively treated as frictionless. In contrast, friction or traction plays

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Nomenclature

a_1	$(2-2\nu^2)s/(\pi E)$
a_2	$(2\nu-1)s/(2\pi G)$
b	auxiliary variable (Eq. (A2))
c	correction (Eqs. (10) or (13))
c_x, c_y	coefficients; either 0 or 1
$c_{z,x}$, $c_{z,y}$	auxiliary variables (Eq. (A3))
$c_{zz,x}$, $c_{zz,y}$	auxiliary variables (Eq. (A4))
c_1, c_2	constants in the fluid mass-density function
d	sum of roughness local heights of the contact counter-surfaces
$d_{z,x}$, $d_{z,y}$	auxiliary variables (Eq. (A5))
$d_{zz,x}$, $d_{zz,y}$	auxiliary variables (Eq. (A6))
D	$R_x + R_y - \sqrt{R_x^2 - x^2} - \sqrt{R_y^2 - y^2}$
D_x, D_y	contact-ellipse semi-axial lengths
E	elastic modulus
f, f_x, f_y	functions (Eqs. (7)–(9))
F	contact normal load (Fig. 1)
G	$E/(2+2\nu)$; shear modulus
h	film thickness (Fig. 1; Eq. (1))
h_c, h_{min}	central and minimum film thickness
H	height of the variator in Fig. 7
$i, j; i_s, j_s$	discrete coordinates of square
m	$10^9\alpha/[5.\ 1\bullet\ln(\eta_0)+49.\ 317]$
n	contact efficiency (Eq. (A15))
N_x, N_y	numbers of partitions in Ox and Oy
p	pressure
P	tangent plane of the contact (Fig. 1)
r	radius of the variator in Fig. 7
r_x, r_y	radii of curvature of the roller
R_x, R_y	radii of curvature
s	side semi-length of square
S	contact area (Fig. 1)
sgn(•)	sign function
u_r, u_d	tangential velocities of the roller and the output disc
U	$(u_r+u_d)/2$; rolling velocity
V	u_r-u_d; nominal sliding velocity
w	normal elastic displacement (Eq. (2))
w_p	pressure-induced normal elastic displacement (Eq. (3))
w_s	normal elastic displacement (Eqs. (6) and (11))
w_x, w_y	traction-induced normal elastic displacements (Eqs. (4) and (5))
x, X, y, Y, z	coordinates

Greek symbols

α	pressure-viscosity coefficient
β	$\sqrt{(x-X)^2+(y-Y)^2}$
γ, δ	coefficients in the τ_L-function
ε	normalized correction (Eq. (15))
ε_{max}, ε_{mean}	maximum and mean value of ε
ζ_k	$j - j_s - (-1)^k/2$ (k=1, 2)
η	$\eta_0 e^{\left\{[\ln(\eta_0)+9.67]\left[(1+5.1\bullet10^{-9}p)^m\right]-1\right\}}$; dynamic viscosity at temperature θ
η_0	dynamic viscosity at temperature θ and atmospheric pressure
η_x, η_y	effective dynamic viscosities
θ	lubricant bulk temperature
λ	relative correction (Eqs. (14) and (16))
λ_p	$(1-\nu^2)p(x, y)/(\pi E)$
λ_x	$(2\nu-1)\tau_{zx}/(4\pi G)$
λ_y	$(2\nu-1)\tau_{zy}/(4\pi G)$
Λ	parameter of the rheological model [8,21]
μ	coefficient of friction
μ_r, μ_d	traction coefficients of the roller and disc (Eq. (A14))
ν	Poisson's ratio
ξ_k	$i - i_s - (-1)^k/2$ (k=1, 2)
ρ	$\rho_0\left(1+\frac{0.6\bullet10^{-9}p}{1+1.7\bullet10^{-9}p}\right)$; fluid mass density
ρ_0	fluid mass density at temperature θ and atmospheric pressure
τ_L	$\tau_0 + \gamma p - \delta\theta$; limiting shear stress
τ_{zx}, τ_{zy}	tangential tractions
$\tau_{zx}^{(r)}$, $\tau_{zy}^{(r)}$	tangential tractions on the surface of the roller
τ_0	constant in the τ_L-function
φ	roller angle (Fig. 7)
ω	angular velocity of the output disc (Fig. 7)

a major role in the assessment of EHL numerical results concerning the minimization of wear and power loss. Tangential tractions also cause tangential displacements, which modify the contact geometry and even the surface velocities employed in the Reynolds equation, particularly in soft contacts [6]. The latter is not explored in the present study but has been incorporated by the author in contact problems where geometrical and kinematical precision is critical such as in modelling the entrapment of micro-particles in concentrated hard contacts [7].

What is of concern in this study is the effect of tangential tractions in the calculation of normal displacements in concentrated contacts. Particularly, this is explored for EHL contacts in terms of the central and the minimum film thickness, the traction coefficient and the contact efficiency. Tangential tractions have been analytically formulated and included in all of the author's published studies since the 1990s, concerning various contact and lubrication problems; for example, traction drives [8], elastomeric hydraulic reciprocating seals [9] and thermoelastoplastic indentation problems [10]. In the present article, analytical equations are derived to approximate the numerical error associated with ignoring the effect of tangential tractions on elastic normal displacements in the case of uniform normal and tangential loading with Amontons' friction laws. It is shown that the error is proportional to the coefficient of friction. Moreover, it is a monotonic rational function of the Poisson's ratio and a nonlinear function of the ratio of the principal dimensions of the contact. The formulation concerns rectangular and elliptical pressurized contacts with either one-directional or bi-directional traction. Additional results show the precise numerical error when tangential tractions are ignored in the much more general case of a rolling-sliding-spinning, elliptical, non-Newtonian, isothermal and smooth EHD contact of a toroidal traction drive. The results show that the central film thickness is unaffected by the omission of tangential tractions and this is analysed mathematically and shown graphically through a parametric analysis in this article. However, a critical error in the calculation of the minimum film thickness is revealed, which may invalidate existing formulae in the literature under some kinematical conditions. For example, there are cases where an EHD film of nearly zero thickness is found whilst published formulae predict a sufficiently thick film, a result with obvious repercussions on wear. Moreover, significant errors on the traction coefficient and contact efficiency are found in some cases, which bring additional concerns over the use of some published formulae and studies that ignore tangential tractions intrinsically.

2. Mathematical analysis

An example of two solids in contact and in relative motion is shown in Fig. 1. The solids are loaded by a normal force, F. The contact may be