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A unified approach for representing fretting and damage at the edges of incomplete and receding contacts

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ABSTRACT

This paper attempts to unify the analysis of fretting and damage at the edges of different classes of contact. Specifically edge asymptotes may be applied, in the same form, to both incomplete and to receding contacts. The benefit of this approach is that experiments carried out on incomplete contacts may be used to predict the life of a wide range of incomplete contact geometries, and also of receding contacts.

1. Introduction

The nucleation of fatigue cracks under conditions of fretting remains something which is not fully understood and, indeed, the micromechanics of the process are likely to remain elusive and also to vary from alloy to alloy. So, although there has been, and continues to be, a lot of interest in modelling this process, usually through some form of critical plane analysis, we advocate a wholly different approach. The basic idea is that a (small) family of local solutions, centred on the edge of the contact, and which implicitly includes a lot of information about the local stress field, including both the stress gradient and polar variation of stress components, is fitted to the edge of the contact. As will be seen, these solutions fully describe the contact tractions, extent of slip, slip displacement, and all other relevant information which might have a bearing on crack nucleation. It follows that, if laboratory tests are carried out in which the history of these local solutions is well specified, both the threshold for infinite life (and, in the case of finite life, the number of cycles taken to nucleate a crack), of a prototype suffering the same history of local solutions must have exactly the same life; the micromechanics of what goes on within the domain of that local solution need not concern us.

Before we can begin the analysis we need to carry out a taxonomy of all possible classes of contact which might be found in any prototype, and, in Fig. 1 we show, in idealised form, the four basic forms of behaviour which might be experienced. It is assumed that the components experiencing fretting are made from metals or alloys, and that the overall loads are such as to maintain an elastic macroscopic stress state in the material: the deformation of the components is therefore small and conventional linear elasticity with no significant rotation of material elements applies. The state of stress varies with a radial coordinate from the contact edge, s, in a characteristic way. First, Fig. 1a shows an incomplete contact (in fact a cylinder pressed onto a flat, but other problems falling into this class include a shallow wedge or, more practically, the dovetail root of a fan blade root), where both bodies may, in the neighbourhood of contact, be idealised by a half plane (or half-space in three dimensions). When half-plane theory is employed we may show from the properties of singular integral equations usually employed to solve contact problems [1] that the contact pressure, $p(s)$, must be locally square root bounded, ie. $p(s) \propto \sqrt{s}$. A similar kind of contact which is incomplete but which may not be modelled using half plane theory, such as the pin in an almost conforming hole (a more practical case of a problem of this class would be the edge of a turbine blade firtree root in a gas turbine) is shown in Fig. 1b. It is *still* the case that locally the contact edge still behaves as a half plane, and hence the contact pressure still decays in a square root bounded manner.

We turn, now, to a complete contact such as the one depicted in Fig. 1c in which an elastic block is pressed into an elastically similar half-plane (a contact occurring in a gas turbine of this class is the spline joint between split shafts). Here, there are a number of possibilities depending on the contact edge behaviour and whether the contact is locally stuck [2,3] but, in every case, the contact pressure will vary as $p(s) \propto s^{\lambda-1}$, where λ is an eigenvalue

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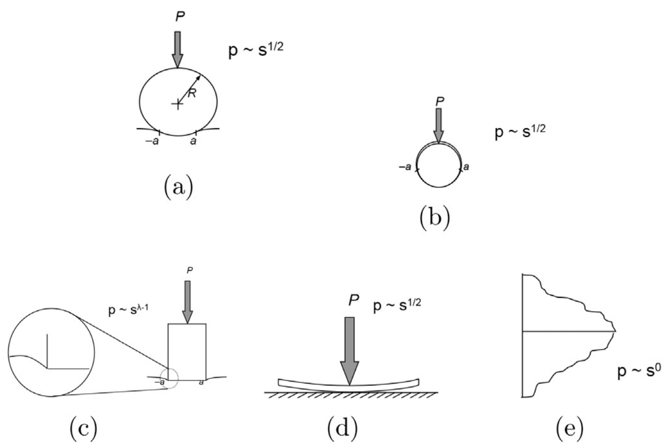

Fig. 1. Fundamental classification of contacts. (a) Incomplete and non-conformal contact, (b) incomplete and conformal contact, (c) complete contact, (d) receding contact, and (e) common edge contact.

of a characteristic equation, parametrically dependent on the local contact angle and coefficient of friction, but almost invariably $0 < \lambda < 1$, so that the contact pressure, moving towards the contact edge, displays a power order singular characteristic. Fig. 1d depicts a receding contact. The most commonly occurring cases are in bolted joints, where the very localised contact pressure beneath the bolt causes the interface to separate away from the point of application of the applied force. These will be described in more detail in a later section, but we note, here, that the contacting surfaces at the point of separation have a common tangent, and so locally these are again very similar in behaviour to an incomplete, half-plane contact, and hence the contact pressure displays square root bounded characteristics. Lastly, in Fig. 1e we show, in idealised form, the edge of a contact formed by two bodies whose edges are aligned, so that the location of the contact edge is defined by both bodies simultaneously. Contacts with a common edge still have a local stress distribution with a characteristic local polar form [4,5], and the contact pressure is finite and non-zero, i.e. $p(s) \propto s^0$. So, it may be seen that complete contacts and those having a common edge exhibit a different edge characteristic from incomplete and receding contacts. We will, in the rest of this paper, restrict ourselves to consideration of contacts displaying square root bounded contact edge behaviour, and show how problems of these classes may be treated uniformly if only the contact edge behaviour, including the presence of local slip, are needed. This is invariably the case when fretting fatigue arises.

2. Basic solutions

The best way to describe contact edge behaviour is to assume, for the time being, that the coefficient of friction, f, is sufficiently high to inhibit all slip. Fig. 2 shows a generic incomplete contact (which need not be Hertzian). If the contact half-width is a the symmetric shear traction, $q(x)$, induced by the application of a shear force Q is given by

$$q(x) = \frac{Q}{\pi\sqrt{a^2 - x^2}}. \tag{2.1}$$

It is rare for contacts to suffer only a net force, and, in most cases, tensions parallel with the free surfaces will be present. If that in the upper body is σ_1 and that in the lower body is σ_2 an antisymmetric shear traction distribution is induced given by

$$q(x) = -\frac{\sigma_0 x}{4\sqrt{a^2 - x^2}}, \tag{2.2}$$

where $\sigma_0 = \sigma_1 - \sigma_2$. Notice that these results are independent of the geometry of the indenter, and that, if we move the origin to a local one positioned at either edge of the contact, positive inwards, Fig. 2a, and then expand these expressions by the binomial theorem and take the dominant term, we see that the local shear traction is given by

$$q(s) = \frac{K_T}{\sqrt{s}}, \tag{2.3}$$

where

$$K_T = \pm\frac{Q}{\pi\sqrt{2a}} + \frac{\sigma_0}{4}\sqrt{\frac{a}{2}}, \tag{2.4}$$

and the +ve sign is taken for the left-hand edge $(x \to a^+)$ and the −ve sign is taken for the right-hand edge $(s \to a^-)$. These expressions display the magnitude of the shear stress singularity at each edge of the contact and show that the application of a differential surface tension is additive to the effects of a shear force at one end of the contact but is subtractive from it at the other. We turn, now, to the solution for the contact pressure. For incomplete contacts this may always be written in the form

$$p(s) = K_N\sqrt{s}, \tag{2.5}$$

and so we define the multiplier $K_N[FL^{-5/2}]$ by

$$K_N = \lim_{s\to 0}\frac{p(s)}{\sqrt{s}}. \tag{2.6}$$

For simple geometries this may be evaluated analytically so that, for example, for a Hertzian contact

$$K_N = \frac{P}{\pi}\sqrt{\frac{8}{a^3}}, \tag{2.7}$$

and for complex profiles where the finite element method is used the value may be abstracted by conventional numerical procedures.[1] Once the values of K's are known, giving the fully adhered contact edge response, the effect of friction may easily be found. This is, of course, loading trajectory dependent and so, here, we will consider the case where first the normal load is applied, and subsequently held constant. By using the Ciavarella–Jäger theorem [7,8] the corrective solution for the shear traction may be found [9,10], and the size of the slip zone for a monotonically increasing set of loads producing advancing slip is determined. The steady state (reversing) slip zone size under reversing loading is also given. This has the result, for the *steady state*, of a slip zone of length d, given by

$$fd = \frac{\Delta K_T}{K_N}, \tag{2.8}$$

where ΔK_T is the range of shear stress intensity, and the shear traction is given by

$$q(s) = fK_N\sqrt{s} = \frac{K_T}{d}\sqrt{s} \quad 0 < s < d \tag{2.9}$$

$$q(s) = \frac{K_T}{d}(\sqrt{s} - \sqrt{s-d}) \quad d < s, \tag{2.10}$$

[1] This is related, we believe, to the quantity, I^s, introduced by Montebello [6], which he refers to as the 'velocity field', and we believe that it is defined by the relationship $\frac{\partial p(x,a)}{\partial a} = \frac{I^s}{\sqrt{a-x}}$. It may easily be shown to be related to the quantity $K_N = 2I^s$.