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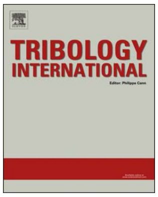

Realistic friction coefficient model between a rolling cylindrical element and a deformable flat surface

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ABSTRACT

A varying friction coefficient model between a rolling cylindrical element and a deformable flat surfaces is predicted. The flat surface is assumed to undergo plastic deformation while sliding under the cylindrical rolling element. Current research models are modified using a technique in which the contact region is analyzed in a piecewise manner from the entry to the exit points and the instantaneous coefficient of friction extracted for each element. The model revealed that friction coefficient is a function of the roll angle, and also that, the function between friction coefficient and the roll angle is not a continuous function within the contact region. This phenomenon is consistent with the physics of contact rolling friction.

1. Introduction

This work focuses on predicting varying friction coefficient between a rolling cylindrical element and flat surfaces. The flat surface is assumed to undergo plastic deformation while sliding under the cylindrical rolling element. The mechanics of friction are complex, and the fundamentals of the phenomenon have been the subject of considerable study [1,2]. Nevertheless, very little is known that would facilitate the formulation of the exact functional relationship between the friction force and the process variables. Several attempts have been made by researchers including Hill [3], Avitzur [4], Lenard [5], Tieu [6], and Abdollahi and Dehghani [7] in investigating the nature of the coefficient of friction in terms of some of the significant parameters, in the rolling process. Friction coefficient are usually in terms of the roll separation force, the radius of the deformed roll, the resistance to deformation, the entry thickness, and the exit thickness. Current work assumes constant coefficient of friction in the contact region. The assumption of a constant coefficient of friction is unrealistic because there is a relative motion between the surface of the part and the rolls which causes sliding, sticking and slipping actions within the contact region. Since the work flow is continuous, there is a gradual change in the relative speed between the rolls and the part. However, there is a point at which the roll and the part speeds are equal. This point is known as the “no slip” point or the “neutral point”. From the entrance of the roll to the “no slip” point, the roll moves faster than the work, while the work moves faster than the roll from the “no slip” point to the exit point. The above phenomenon reflects a varying friction coefficient within the contact region and not a constant friction coefficient as other researchers have alluded to. This work seeks to provide a realistic estimation of instantaneous friction coefficient within the contact region for slab rolling operation. In this study, the “rule of thumb” and other current methods have been improved using a new modeling technique where the contact region is incrementally analyzed to extract the coefficient of friction. Using the exit thickness as the reference, incremental thicknesses are evaluated using elemental roll angles. The result from the incremental thickness is substituted into a friction coefficient model which is modified to evaluate friction coefficient at each incremental point. Consequently, friction coefficient is determined as a function of roll angle.

2. Current modeling results

One of the most popular models is given by Hill [3] as in Eq. (1). where,

$$\mu = \frac{\frac{P_r}{\bar{\sigma}\sqrt{R'\Delta h}} - 1.08 + 1.02\left(1 - \frac{h_{exit}}{h_{entry}}\right)}{1.79\left(1 - \frac{h_{exit}}{h_{entry}}\right)\sqrt{\frac{R'}{h_{entry}}}} \tag{1}$$

P_r is the roll separating force per unit width, $\bar{\sigma}$ is the average plane strain flow strength in the pass and R' is the radius of the flattened roll [5]. Examination of Hill's formula shows that the friction coefficient (μ) is in terms of the entry and exit the thickness, roll separation force,

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Nomenclature

h_0	Initial Slab Thickness (mm)
h_f	Final Slab Thickness (mm)
R_0	Roll Radius (mm)
$d\theta$	Change in Roll Angle (radians)
θ	Roll Angle (radians)
F	Friction Force (Ibf)
P	Normal Pressure (Ibf)
σ	Normal Stress (N/mm^2)
μ	Friction Coefficient
P_r	Roll Separation Force (N)
$\bar{\sigma}$	Average Flow Stress (N/mm^2)
R'	Radius of Flattened Roll (mm)
σ_y	Yield Strength (N/mm^2)
σ_{xb}	Back Tension (N/mm^2)
σ_{xf}	Front Tension (N/mm^2)
N_r	Length of Rolled Part (km)

average flow stress, and deformed roll radius. Since the pressure distribution varies within the contact region, the roll separation force is expected to vary, and so will the flow stress. Consequently, averaging the flow stress might not be a true reflection of the rolling operation. Moreover, the roll separation force and the deformed roll radius are input parameters that can only be determined through measurement. Avitzur sought to address the challenge in determining the roll separation force and the deformed roll radius in Hill's friction coefficient formula. He used the energy method to derive an expression for estimating the friction coefficient in terms of the rolling geometrical parameters and material properties. Avitzur's expression which is shown in Eq. (2) is in terms of the entry and exit thicknesses, the roll radius, the back and front tensions, and the yield strength of the work material.

$$\mu = \frac{\frac{1}{2}\sqrt{\frac{t_f}{R_0}}\left\{\ln\left(\frac{t_0}{t_f}\right) + \frac{1}{4}\sqrt{\frac{t_f}{R_0}}\sqrt{\frac{t_0}{t_f} - 1} + \frac{(\sigma_{xb} - \sigma_{xf})}{[2/\sqrt{3}\sigma_0]}\right\}}{\left\{\left(\ln\frac{t_0}{t_f} - 1\right)\frac{\sigma_{xf} - \sigma_{xb}}{\frac{2}{\sqrt{3}}\sigma_0\sqrt{\frac{t_0}{t_f} - 1}} - \left[\frac{1}{\frac{2}{\sqrt{3}}\sigma_0}\left(\sigma_{xb} - \frac{\sigma_{xf} - \sigma_{xb}}{\frac{t_0}{t_f} - 1}\right) - 1\right]\tan^{-1}\sqrt{\frac{t_0}{t_f} - 1}\right\}} \tag{2}$$

Avitzur succeeded in dealing with the difficulty in calculating the roll separation force and the deformed roll radius in Hill's formula. He also introduced back and front tensions in his estimation, and assumed a constant flow stress within the contact region. Unfortunately, the estimation of friction coefficient within the contact region also resulted in a constant value [4]. Tieu [6,8] investigated the relationship of the factors involved in the roll force calculation based on the Hill's friction coefficient models. The deformation resistance and the friction coefficient were determined simultaneously by minimizing the error of the measured and calculated rolling forces using nonlinear least square optimization algorithm. The general equation proposed to describe the friction coefficient was $\mu = (a + bh + cr)\frac{d}{1 + eN_r}$, where a, b, c, d, and e are coefficients, N_r, h, and r are the length of coil (length of rolled part), the exit thickness, and the reduction respectively. The optimization method used Steel Material, N_r=3000 km, R=270 mm, $\Delta h = h_0 - h$=1.96 mm, $r = \frac{h_0 - h}{h}$ as the input parameters. The resulting friction coefficient model under these conditions is shown in Eq. (3).

$$\mu = (0.01469 + 0.0298\,h + 0.00167\,r)\frac{1.09979}{1 + 0.000929\,N_r} \tag{3}$$

From the model, friction coefficient is in terms of the geometrical parameters. According to the authors, the material resistance which is given in terms of the length of coil accounts for the material property. Tieu's model is also in terms of the entry and the exit thickness. The friction coefficient within the contact region also resulted in a constant value [8].

The existing friction coefficient models have the following characteristics: (1) they use only the entry and the exit thicknesses of the work part, (2) they have input parameters which are difficult to obtain, (3) they approximate flow stress to be constant within contact region, and (4) they result in a constant coefficient of friction value. There is, therefore, the need to continue to search for a friction coefficient model within the contact region that obeys the laws of friction and is easier to estimate. This work investigates the friction coefficient as a function of roll angle and seeks to address the above needs by modifying Avitzur and Tieu's models. The next heading discusses modified modeling technique and the methodology used in establishing the realistic friction coefficient models.

3. Modified modeling technique

The current modeling techniques assume several constant parameters in the contact region and estimate the friction coefficient using only the entry and exit conditions. Fig. 1 illustrates the characteristics of the current modeling input parameters where h_0 is the entry slab thickness, h_f is the exit slab thickness, R_0 is the roll radius, and θ_{max} is the maximum included angle.

This work proposes to section the contact region into piecewise strips of varying thicknesses and then evaluates the friction coefficient at each strip. Since the centers of the two rollers are fixed, the exit thickness is used as the reference. Figs. 2 and 3 illustrate the proposed modified modeling technique. The roll angle is also measured from the reference line and its incremental direction is also shown in Fig. 3.

The symbols h_i and h_{i+1} represent the exit and entry thicknesses respectively for the ith elemental strip. Given the roll radius (R_0), the entry thickness (h_n) or exit thickness (h_0), and the maximum reduction (r), the roll included angle (roll bite angle) can be calculated from the geometry of the arc contact. For each incremental angle, the corresponding incremental thickness can be evaluated using Eq. (4). The present friction coefficient models are then developed based on the elemental strips.

$$h_{i+1} = h_i + 2R(1 - \cos\theta_i) \tag{4}$$

3.1. Modification of Tieu's model

Tieu's friction coefficient model was modified to assume the difference equation form as shown in Eq. (5).

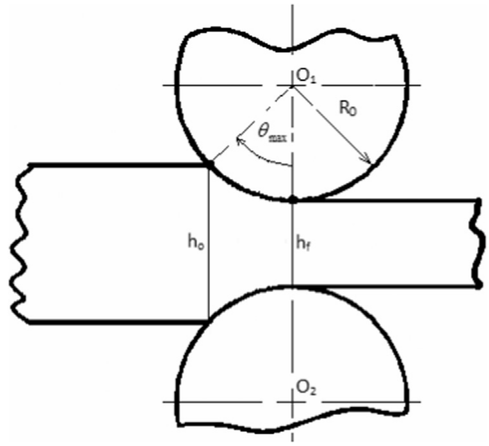

Fig. 1. Current modeling technique.