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An analysis of generated fractal and measured rough surfaces in regards to their multi-scale structure and fractal dimension

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ABSTRACT

This work studies the methods used to extract fractal dimensions from surface profiles and the applicability of fractals to measured surfaces. The work used generated surfaces to first evaluate these methods, and later applies the methods to measured surfaces. Two methods for generating surface profiles are used. The fractal dimension values of all generated profiles are calculated by four methods, then the results are compared. It is also found that self-similarity occurs at different fractal dimensions for the power spectrum and the Weierstrass-Mandelbrot generation methods. The analysis indicates that real measured rough surfaces are not easily represented as perfect fractals as researchers and engineers often assume.

1. Introduction

The study of natural surfaces has been a valuable topic for centuries. In fact, scientists have extended the study of surfaces to coastlines on the macro scale [1] and cell surfaces [2] on the micro scale. Surface characterization is very important to practical applications: the electrical connectors [3], the mechanical seals [4] and the articular cartilage [5]. Researchers [6–8] also mentioned that the roughness of road surfaces will have an impact on the vehicle tires which will affect fuel economy. When studying surfaces, the method used to characterize them is very important. Surface characterization then became a significant technique in studying the surfaces. The surface characterization of rough surfaces impacts many important physical phenomena such as sealing, wear, friction and electrical contact.

During the study of rough surfaces, engineers have created surfaces by many finishing technologies, such as casting, turning, milling, grounding, polishing, lapping and so on. It is also found that the roughness of surfaces can be controlled and improved through the manufacturing process and machining of the components [9,10], so that surface fatigue and wear between two surfaces, along with some other surface phenomena can be controlled. The development of surface processing technology further indicates the important of surface characterization. Álvar et al. [11] mentioned that machining using end-milling tools or turning ones can cause regular patterns, the periodic structures occurring from man-made regular patterns will affect and be captured by fractal and especially spectral analysis. For instance, a spectrum of these surfaces should show a high peak or amplitude on the scale of these features. Therefore, surface characterization is also of great economic importance to various industries and applications.

Initially, only Euclidean geometry was employed for surface characterization. However, this kind of geometry has many limitations because it can be difficult to compute the multi-scale roughness of a real world geometry. Mandelbrot was the first person who pointed out that the fractal geometry seems much more suitable for describing the natural world, which is inherently rough on many scales [12]. Mandelbrot also defined a fractal as “a shape made of parts similar to the whole in some way” [13].

When using the fractal geometry to characterize rough surfaces, the most important parameter is the fractal dimension, D. It describes the space occupancy of an object and can be used to quantify the roughness of an object. Non-fractal Euclidian geometries have integer values for their fractal dimensions, like a line has a fractal dimension equal to one (D=1), the fractal dimension of a plane is two (D=2) and a space has fractal dimension equals to three (D=3). Fractal geometry can work with objects that are non-Euclidean because they have non-integer dimensions. The non-integer fractal dimension means an object is ‘in between’ these geometries due to features or roughness along the border of the object. For instance, one could envision that a line could approach the geometry of a plane as its roughness increases to become very large. Hence, roughness can be said to cause an object to have a dimension in between these geometries.

Since the fractal dimension is an important and popular parameter in characterizing fractal rough surfaces, there are many methods to calculate it or extract it from a surface, like the classic Richardson plot

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Nomenclature

B	asperity aspect ratio or Fourier series slope
D	fractal dimension
G	fractal roughness
K	wave number
K_r	roll-off wavenumbers
K_s	upper cutoff wavenumbers
L	length of the profile
n	fractal scale index
n_1	lowest cutoff frequency index
n_2	highest cutoff frequency index
R_q	root mean square roughness
s	sampling resolution
z	profile height
τ	distance in the variogram method
γ	fractal scaling parameter
Δ	amplitude
λ	wavelength

[14], the compass method [1,15], [16], the variogram method [17], the roughness-length method [18], the RMS-COR method [19] and the power spectral density (PSD) method [17,20]. Note that the variogram method is the same as the structure function method [21]. In this paper, the authors picked four different methods (the roughness-length method, the box-counting method, the variogram method and the power spectral density method) to calculate the fractal dimension value. The reason for choosing the roughness-length method and PSD method is because they are all widely used and very popular for calculating the fractal dimension. Since the box counting method is widely used but listed to calculate the fractal dimension of self-similar profiles, it is also considered. We can also test if this method can be used in calculating the fractal dimension of self-affine profiles and measured surfaces that might be self-similar as suggested by Jackson [22]. As to the variogram method, it is used to a lesser extent, but suggested by an expert in the field, so the effectiveness of this method on our profiles is tested.

In the roughness-length method, the relationship between RMS roughness and the measured window length of the profile is plotted on a log-log scale. The slope of the plot then can be used to calculate the fractal dimension. In the box-counting method, the surface profile is covered by an array of identical boxes, whose size will be changed by a power of 2 and the number of covered boxes are counted. The fractal dimension is exactly the slope of the log-log plot between the box size and box number. For the power spectral density method, the auto-correlation function (ACF) is calculated first and the FFT algorithm is then implemented on the ACF to obtain a power spectral density. Note that the maximum frequency (inverse of the upper wavelength) is used to normalize the power spectrum values, resulting in power spectral density (PSD). By taking the logarithm of the power spectral density equation, the obtained slope is used to calculate the fractal dimension value. More detailed descriptions of these three methods can be found in previous work [23]. For the variogram method, a large number of pairs of points separated by different lateral distances (τ) along the profile are chosen, then the difference in height between two positions are calculated (e.g., z values). By calculating the sum of squares of all the height differences and then computing the average value of the sum, the fractal dimension can be derived from the log-log plot of the average values verses distances between the point pairs (τ). According to [17,24], the expression of the variogram method can be written as:

$$s(\tau) = \frac{1}{N(\tau)} \sum_{i=1}^{N(\tau)} [z(x_i + \tau) - z(x_i)]^2 \quad (1)$$

where $s(\tau)$ is defined as the variogram, and $N(\tau)$ is the number of point pairs separated by a distance τ.

Due to the complex structures of actual multi-scale rough surfaces, many simplified methods are developed to consider multi-scale and fractal profiles, such as the successive random addition method [25], the midpoint displacement method [26], the Fourier based filtering algorithm [27,28] and the Weierstrass-Mandelbrot (W-M) function method [29,30]. The reason we call them “simplified” methods is just because these methods use equations to simplify the complex structures of rough surfaces to simple structures. In this study, the inverse Fourier transform based on a prescribed Power Spectrum Density (PSD) method and the Weierstrass-Mandelbrot (W-M) function method are only considered and used to generate surface. In the inverse Fourier transform based on a prescribed Power Spectrum Density (PSD) method (PSD-iFFT), the power spectral density (PSD) is calculated first [27], then the surface profile is generated from the PSD. In contrast, the W-M function method generates a fractal profile by adding non-fractal sinusoidal shapes together [29] as prescribed by the Weierstrass-Mandelbrot function. The major difference between these methods is that the PSD-iFFT method generates a surface profile from a PSD that is an arithmetic series, while the W-M function is a geometric series. The geometric series has terms that are spaced over wavelengths logarithmically. The Fourier series is a popular example of an arithmetic series and is used in this work for the PSD-iFFT.

There are also other methods to characterize the multi-scale structure of surfaces. According to Jackson [31], the slope of the asperity aspect ratio (B) versus wavelength (λ) (i.e.$dB/d\lambda$) is an effective parameter for characterizing multi-scale surface structure and its degree of self-affinity. The asperity aspect ratio (B) indicates the ratio of the amplitude (Δ) of the multi-scale surface at a scale to the wavelength (λ) of that scale.

For a perfectly self-similar rough surface, when B is plotted versus λ, it results in a straight horizontal line, which means no matter how the scale changes, B is a constant over the scales and the $dB/d\lambda$ value is zero. However, for self-affine surfaces, $dB/d\lambda$ will have a non-zero value and B will vary linearly as a function of wavelength. Of course, this methodology assumes that the surface is composed of wavy functions.

The objective of this work is not to promote or demonstrate the use of fractals, rather, we are analyzing the effectiveness of using fractals to characterize real surfaces. We are also attempting to answer the following question: Which methods are the most reliable in extracting the fractal quantities from a surface profile (i.e. fractal dimension)? These methods are analyzed by comparing the extracted values to the known values used to generate ideal fractal surface profiles. Once these methods are identified, we use them to characterize measured rough surface profiles and test their effectiveness. We then quantify the self-affinity of the measured rough surfaces using $dB/d\lambda$, and test if the predicted fractal dimensions are in agreement. From the results of our analysis, one can choose an appropriate method to calculate the fractal dimension value of the profile and can also help people to understand the multi-scale nature of actual measured surfaces.

2. Characterization of generated surface profiles

The objective of this section is aimed at analyzing the effectiveness of various techniques on extracting the fractal dimensions. This is done by using the surface generation techniques to create rough surface profiles with a known fractal dimension. Whether self-similarity occurs at the same value for the generated surface profiles by these two methods is also tested.