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Incompressible moving boundary flows with the finite volume particle method

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ABSTRACT

Mesh-free methods offer the potential for greatly simplified modeling of flow with moving walls and phase interfaces. The finite volume particle method (FVPM) is a mesh-free technique based on interparticle fluxes which are exactly analogous to intercell fluxes in the mesh-based finite volume method. Consequently, the method inherits many of the desirable properties of the classical finite volume method, including implicit conservation and a natural introduction of boundary conditions via appropriate flux terms. In this paper, we describe the extension of FVPM to incompressible viscous flow with moving boundaries. An arbitrary Lagrangian–Eulerian approach is used, in conjunction with the mesh-free discretisation, to facilitate a straightforward treatment of moving bodies. Non-uniform particle distribution is used to concentrate computational effort in regions of high gradients. The underlying method for viscous incompressible flow is validated for a lid-driven cavity problem at Reynolds numbers of 100 and 1000. To validate the simulation of moving boundaries, flow around a translating cylinder at Reynolds numbers of 20, 40 and 100 is modeled. Results for pressure distribution, surface forces and vortex shedding frequency are in good agreement with reference data from the literature and with FVPM results for an equivalent flow around a stationary cylinder. These results establish the capability of FVPM to simulate large wall motions accurately in an entirely mesh-free framework.

1. Introduction

In this article we describe a validated development of the mesh-free finite volume particle method (FVPM) for incompressible flow around moving bodies. Mesh-free methods for computational fluid dynamics (CFD) represent the fluid with a set of moving nodes or particles rather than with a fixed mesh of nodes with predefined connectivity. Mesh-free methods are particularly suited to flows with free surfaces, moving walls and multiple phases because interfaces can be treated without remeshing or special geometric treatments. Furthermore, they have the potential to reduce the cost of expert human effort required for mesh generation.

Smoothed particle hydrodynamics (SPH) is the most mature and widely applied mesh-free method for CFD. SPH, proposed independently by Gingold and Monaghan [1] and Lucy [2], is a fully Lagrangian technique in which the particles have fixed mass, and local conservation is enforced through pairwise symmetric particle interactions. Gradient approximations for the flow variables at each particle are computed on the basis of a smoothed interpolation process. SPH was originally developed for unbounded problems in astrophysics, but has seen extensions to industrially relevant applications. A recent review of SPH is given by Monaghan [3]. The method has yielded accurate predictions of challenging realistic problems including complex 3D unsteady free-surface flows [4].

Boundary conditions in SPH are typically enforced through fictitious particles on the boundary and/or outside the fluid domain. This approach is difficult to generalise to arbitrary geometries. The basic SPH gradient approximation is not exact for constant-valued functions (i.e. not zero-order consistent) [5] and numerical error does not necessarily vanish as the particle size tends to zero [6]. The consequences of this behaviour are not fully understood. All particle-based methods incur relatively high computational costs because of the use of a large computational stencil, which must be reconfigured after every particle position update. The cost is compounded by the facts that the initial particle distribution evolves with the flow, and that SPH-like methods suffer degraded convergence and/or conservation properties in the presence of non-uniform particle distributions. Consequently, it is not straightforward to employ a heterogeneous particle distribution to allocate computational effort economically to the spatial regions where it is most needed.

These problems have motivated the development of mesh-free particle schemes with improved accuracy, simpler boundary condition implementations, and a capacity for non-uniform particle distributions. These include the corrected SPH schemes of Randles and Libersky [7] and Bonet and Lok [8], which ensure first-order consistency at the expense of conservation. Other authors have developed flux-based formulations including the smooth volume

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integral conservation method of Ismagilov [9], the Riemann-SPH schemes of Vila [10] and Monaghan [11], and the finite volume particle method (FVPM) of Hietel et al. [12]. In this article we focus on the FVPM, which maintains conservation even when particle size and particle overlap are not uniform.

In FVPM, the fluid is represented by a set of particles, which are associated with normalised, overlapping, compactly supported kernel functions. The particles are viewed as discrete volumes for which the governing equations are written in integral form, weighted by test functions. Particle interactions are defined in terms of fluxes, which are weighted depending on the overlap of the kernel supports. The FVPM is closely analogous to the classical finite volume method (FVM) and inherits many of its desirable properties, including exact conservation and a natural introduction of boundary conditions via appropriate flux terms. Furthermore, well-established developments for traditional CFD methods, such as upwind flux formulations, may be used in the FVPM without modifications. The basic FVPM scheme has been extended to incorporate adaptive variation of the particle support radius [13], moving boundaries in inviscid compressible flow [14,15], a projection technique for incompressible flow [16,17], and higher-order accuracy and viscous flow [18].

Many important and challenging applications of fluid dynamics are characterised by incompressible viscous flow with moving walls or interfaces. These include flow in blood vessels, medical devices and marine systems. In this article, we describe a development of the finite volume particle method with pressure projection for incompressible viscous flow with moving boundaries (Section 2). The mesh-free character of the FVPM is exploited in an arbitrary Lagrangian–Eulerian (ALE) framework to handle the discretisation near the moving body. In addition, we introduce an improved formulation for the computation of particle volume, and exploit non-homogeneous particle distribution to enhance the efficiency of the method. The method is first validated for incompressible viscous flow in a lid-driven cavity (Section 3.1). The capability to model flow with moving walls is validated by simulating flow over a cylinder translating relative to domain boundaries at Reynolds numbers from 20 to 100 (Section 3.2). Results are compared with data for a stationary cylinder, as well as results from the literature.

2. The finite volume particle method

2.1. Governing equations

In the present work, we consider the application of FVPM to viscous incompressible flow. The governing continuity and momentum equations are written as

$$\nabla \cdot \mathbf{u} = 0 \tag{1}$$

and

$$\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot (\mathbf{F} - \mathbf{G}) = 0, \tag{2}$$

where $\mathbf{u} = (uv)^T$ is the 2D velocity vector, $\mathbf{U} = \rho\mathbf{u}$, ρ is the density and t is time. $\mathbf{F} = (\rho\mathbf{u} \otimes \mathbf{u} + p\mathbf{I})$ represents the inviscid flux, where $\mathbf{I}$ is the $D \times D$ identity matrix, and D denotes the number of dimensions. The two-dimensional incompressible viscous flux is given by

$$\mathbf{G} = \begin{pmatrix} 2\mu\frac{\partial u}{\partial x} & \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & 2\mu\frac{\partial v}{\partial y} \end{pmatrix}, \tag{3}$$

where μ is the dynamic viscosity.

2.2. The FVPM formulation

The FVPM was originally derived by Hietel et al. [12]. The formulation is outlined briefly here. In FVPM, the fluid is represented by a set of N particles. These particles are defined by compactly supported, overlapping test functions ψ_i of the form

$$\psi_i(\mathbf{x},t) = \frac{W_i}{\sum_{j=1}^{N} W_j}, \tag{4}$$

where $W_i = W(\mathbf{x} - \mathbf{x}_i(t), h)$ is a compactly supported kernel function for particle i, centred at $\mathbf{x}_i$. The compact support radius is $2h$, where h is called the smoothing length, in keeping with the SPH convention. The denominator normalises the kernel function to ensure that the test functions form a partition of unity, i.e.

$$\sum_{i=1}^{N} \psi_i(\mathrm{x},t) = 1. \tag{5}$$

Each particle is associated with a volume

$$V_i = \int_\Omega \psi_i d\mathbf{x}, \tag{6}$$

and a discrete value of any field variable ϕ

$$\phi_i = \frac{1}{V_i}\int_\Omega \phi\psi_i d\mathbf{x}, \tag{7}$$

which is a weighted average of ϕ over the domain Ω. ϕ_i is associated with the particle barycentre $\mathbf{b}_i$, defined as

$$\mathbf{b}_i = \frac{1}{V_i}\int_\Omega \mathbf{x}\psi_i d\mathbf{x}. \tag{8}$$

Denoting as $\mathbf{F}_{ij}$ an approximation for the Eulerian inviscid flux $\mathbf{F}$ between particles i and j, the ALE inviscid flux is $\mathbf{F}_{ij} - \overline{\mathbf{U}}_{ij}\overline{\dot{\mathbf{x}}}_{ij}$, where the $\overline{\mathbf{U}}_{ij}\overline{\dot{\mathbf{x}}}_{ij}$ term is the convection due to the particle motion. The particle velocity $\dot{\mathbf{x}}$ is not necessarily equal to the material velocity $\mathbf{u}$. Following Teleaga and Struckmeier [14], $\overline{\mathbf{U}}_{ij}$ and $\overline{\dot{\mathbf{x}}}_{ij}$ are defined as the averages $\frac{1}{2}(\mathbf{U}_i + \mathbf{U}_j)$ and $\frac{1}{2}(\dot{\mathbf{x}}_i + \dot{\mathbf{x}}_j)$ respectively. Introducing $\mathcal{F}(\mathbf{U}_i, \mathbf{U}_j)$ to denote a numerical approximation to $\mathbf{F}_{ij} - \overline{\mathbf{U}}_{ij}\overline{\dot{\mathbf{x}}}_{ij}$, Hietel et al. [12] derived the semi-discrete form of the FVPM for inviscid flow as

$$\frac{d}{dt}(V_i\mathbf{U}_i) = -\sum_{j=1}^{N} \beta_{ij}\left(\mathcal{F}\left(\mathbf{U}_i, \mathbf{U}_j\right)\right) - \beta_i^b \mathcal{F}_i^b, \tag{9}$$

where

$$\beta_{ij} = \int_\Omega \frac{\psi_i \nabla W_j - \psi_j \nabla W_i}{\sum_{k=1}^{N} W_k} d\mathrm{x} \tag{10}$$

are interaction vectors which weight the flux exchanged between particle i and each of its neighbours j. The interaction vector β_{ij} between particles i and j is analogous to the cell interface area vector which weights intercell fluxes in the classical finite volume method [19]. Eq. (10) is typically evaluated by numerical integration. In Eq.(9), β_i^b is the particle-boundary interaction vector and $\mathcal{F}_i^b$ is an approximation for the boundary flux (boundary conditions are discussed in full in Section 2.7).

The appearance of the particle volume inside the time derivative in Eq. (9) means that an additional equation is required for the rate of change of the particle volume. Hietel et al. [12] show that this can be obtained by differentiating Eq. (6) with respect to time, yielding

$$\frac{d}{dt}V_i = \sum_{j=1}^{N} \left(\gamma_{ij} \cdot \dot{\mathbf{x}}_j - \gamma_{ji} \cdot \dot{\mathbf{x}}_i\right), \tag{11}$$