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Response variability of laminate composite plates due to spatially random material parameter

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ABSTRACT

Young's modulus is at the center of attention in the stochastic finite element analysis since the parameter plays an important role in determining structural behavior. However, the other material parameter of Poisson's ratio is another independent material parameter that governs the behavior of structural systems. Accordingly, the independent estimation of the influence of this parameter on the uncertain response of a system is of importance from the perspective of stochastic analysis. To this end, we propose a formulation to determine the response variability in laminated composite plates due to the spatial randomness of Poisson's ratio. To filter out the independent contribution of random Poisson's ratio, a decomposition of the constitutive matrix into several sub-matrices by using the Taylor's expansion is needed, which makes the random Poisson's ratio simple enough to be included in the formulation. To validate the adequacy of the proposed formulation, several examples are chosen and the results are compared with those given by Monte Carlo analysis. By means of the formulation suggested here, it is expected that an extension of the formulation to include the effect of correlations between random Poisson's ratio and other structural and/or geometrical parameters will be achieved with ease, resulting in a more practical estimation of the response variability of laminated composite plates.

1. Introduction

Intrinsically, the structural materials from general structural steel and concrete in civil engineering applications, which are assumed to be isotropic, to composite materials have randomness in their material properties, such as the elastic modulus, shear modulus, the Poisson's ratio. In particular, the composite materials for which the fibers and matrix materials are combined to complete a lamina, the processes of lay-up and curing are relatively complex when compared with the conventional isotropic materials. In particular, the strength of the composites is known to be affected by the fiber volume fraction [1], which is random in nature. For these reasons the possibility of spatial uncertainty in the material properties of composites is expected to be high. In this respect, the stochastic evaluation of response variability on the composite materials is highly demanding.

In the area of the stochastic finite element analysis, research topics include the investigation of the effect of not only the spatial randomness of material properties but also the geometrical parameters on the response variability [2–6] of various structural types built with conventional materials. Developments, however, linked to the analysis of composite structures with random material properties are relatively limited. Due to the complexity in the mathematical expressions for composite materials, which makes the formulations for the non-statistical approach difficult, the Monte Carlo simulation (MCS) approach has been employed for a variety of stochastic problems including free vibration analysis [7], the effect of random geometrical parameters assumed as distributed in a Gaussian way [8]. Some research works have focused on the cracks and failure of random composites [9]. Regarding the failure probability or reliability, Onkar et al. [10,11], Cassenti [12], Kam et al. [13] and Lekou and Philippidis [14] have demonstrated the effect of random material properties. Hilton and Yi [15] deal with the topic on the delamination of composites. The spectral version of the stochastic FEM methodology has been proposed by Chen and Guedes Soares [16] for laminated composite plates assuming the elastic and shear moduli as Gaussian random fields.

In this study, we restrict the randomness only to the Poisson's ratio in order to investigate the sole effect of this material parameter on the response variability of laminated composite structures. For composite materials, a reciprocal relation exists in between the material constants along mutually perpendicular material axes because of the symmetry in the compliance matrix. Using this feature and then employing the Taylor's expansion on the typical

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fraction form of constitutive relations, we construct the constitutive matrix in the power series expansion form, which enables us to complete a formulation for stochastic finite element scheme for composite materials. The suggested scheme is applied to various composite plates, and the results given by the proposed scheme are compared with the MCS results performed in this study. Some qualitative comparisons are also made if possible employing the results in the literature.

2. Introduction of randomness

2.1. Randomness in Poisson's ratio

From the general concept of randomness in stochastic mechanics, all the material parameters involved in structures have two components: the mean and the deviator. The deviator part is assumed as a function of position, having a specific probabilistic distribution. If we apply the mean operation on a random parameter, the deviator part vanishes and only the mean remains. Accordingly, a parameter p can be assumed to have the following mathematical expression:

$$p(\mathbf{x}) = \overline{p}[1 + f_p(\mathbf{x})], \quad \mathbf{x} \in D_{str}, \tag{1}$$

where, $\overline{p}$ denotes the mean of the parameter $p(\mathbf{x})$, $f_p(\mathbf{x})$ the stochastic field function having zero-mean and D_{str} the domain of the structure. In case of the coefficient of variation (CoV) of the parameter p, the following holds:

$$CV_p = \frac{\sigma_p}{\overline{p}} = \sigma_f, \tag{2}$$

which shows the CoV of a random parameter is equal to the standard deviation of the stochastic field $f(\mathbf{x})$ of the random parameter, which describes the deviator part of the random parameter in the spatial domain.

In expressing the randomness of the Poisson's ratio, the general expression of (1) can be adopted, and the following is assumed:

$$\nu(\mathbf{x}) = \overline{\nu}[1 + f_\nu(\mathbf{x})], \quad \mathbf{x} \in D_{str}. \tag{3}$$

The stochastic field function is assumed to have values $-1 + \delta_f < f_\nu(\mathbf{x}) < 1 - \delta_f$ with $0 < \delta_f < 1$ to avoid the occurrence of a non-physical negative material constant. In this study, the probabilistic distribution of random Poisson's ratio is assumed to follow the Gaussian distribution, which we can assume for a random parameter if the coefficient of variation is relatively small [7,17] around 0.1, and the characteristics of the stochastic field is assumed as not dependent on the position, i.e., homogeneous. In the case of the composite materials, the Poisson's ratio has two different values depending on two material directions. This issue, including the reciprocal character of the Poisson's ratio relative to the two elastic moduli in each material direction, is addressed in the following section.

2.2. Stress–strain relation with random Poisson's ratio

Following the conventional notations and reducing the dimension of material under consideration into that of in-plane, i.e., in the state of plane-stress, the stress–strain relation for (k)th lamina in the principal material coordinates is given as follows:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & \\ Q_{12} & Q_{22} & \\ & & Q_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_6 \end{Bmatrix}^{(k)},$$
$$\begin{Bmatrix} \sigma_4 \\ \sigma_5 \end{Bmatrix}^{(k)} = \begin{bmatrix} Q_{44} & \\ & Q_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_4 \\ \varepsilon_5 \end{Bmatrix}^{(k)}. \tag{4}$$

Each coefficient can be determined in terms of engineering material constants as

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12}E_2}{1 - \nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}},$$
$$Q_{44} = G_{23}, \quad Q_{55} = G_{13}, \quad Q_{66} = G_{12}. \tag{5}$$

In the above expressions, Poisson's ratios ν_{12} and ν_{21} follow the reciprocal relations (with no sum on i, j):

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j}, \tag{6}$$

which enable us to make one of the Poisson's ratios be expressed by the other in terms of elastic moduli for two material directions. Therefore, adopting Eq. (6) for Poisson's ratio ν_{12}, it is possible for the coefficients Q_{11} in Eq. (5), for example, to be modified as follows:

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}} = E_1 \frac{1}{1 - r^2\nu_{12}^2}, \tag{7}$$

where, r^2 is given as the ratio of one elastic modulus to the other, $r = \sqrt{E_2/E_1}$. Usually, r is very small for composite materials [18]. As seen in Eq. (7), the coefficients Q_{11}, Q_{12}, Q_{22} are given in the form of $1/(1 - x)$, which can be expanded using Taylor's expansion formula resulting in $1/(1 - x) = \sum_{k=0}^{\infty} x^k$. Therefore, by substitution of the general expression for a random Poisson's ratio of $\nu_{12}(x) = \overline{\nu}_{12}[1 + f_\nu(x)]$ into Eq. (7) and employing Taylor's expansion, we get the expanded form of coefficient as follows (refer to Appendix I):

$$Q_{11} = E_1 \frac{1}{1 - r^2\nu_{12}^2} = E_1 \left[1 + \sum_{l=0}^{\infty} \left\{ \sum_{k=1,(2k \geqslant l)}^{\infty} (r\overline{\nu}_{12})^{2k} \binom{2k}{l} \right\} f_\nu^l \right], \tag{8}$$

where, $\overline{\nu}_{12}$ and $f_\nu(\mathbf{x})$ denote the mean and the stochastic field function of Poisson's ratio ν_{12}, respectively. In an analogous way, the other terms can be established in a similar form as follows (refer also to Appendix I):

$$Q_{22} = E_2 \frac{1}{1 - r^2\nu_{12}^2} = E_2 \left[1 + \sum_{l=0}^{\infty} \left\{ \sum_{k=1,(2k \geqslant l)}^{\infty} (r\overline{\nu}_{12})^{2k} \binom{2k}{l} \right\} f_\nu^l \right],$$
$$Q_{12} = E_2 \frac{\nu_{12}}{1 - r^2\nu_{12}^2} = E_2 \left[\sum_{l=0}^{\infty} \left\{ \sum_{k=1,(2k-1 \geqslant l)}^{\infty} r^{2(k-1)} \overline{\nu}_{12}^{(2k-1)} \binom{2k-1}{l} \right\} f_\nu^l \right]. \tag{9}$$

In obtaining Eqs. (8) and (9), the binomial theorem is applied, which enables us to derive the expressions for constant terms of respective power stochastic field functions f_ν^l. As seen in Eqs. (8) and (9), we can note that the formulae for constants of Q_{11} and Q_{22} are equal to each other, while the constants for Q_{12} is slightly different from the other two.

Symbolically, we can rewrite Eqs. (8) and (9) as follows:

$$Q_{11} \cong E_1(\beta_0 + \beta_1 f_\nu + \beta_2 f_\nu^2 + \beta_3 f_\nu^3 + \beta_4 f_\nu^4) = E_1 \beta_i f_\nu^i,$$
$$Q_{22} \cong E_2(\beta_0 + \beta_1 f_\nu + \beta_2 f_\nu^2 + \beta_3 f_\nu^3 + \beta_4 f_\nu^4) = E_2 \beta_i f_\nu^i,$$
$$Q_{12} \cong E_2(\gamma_0 + \gamma_1 f_\nu + \gamma_2 f_\nu^2 + \gamma_3 f_\nu^3 + \gamma_4 f_\nu^4) = E_2 \gamma_i f_\nu^i, \tag{10}$$

Even though the theoretical expressions are given as infinite series, we can truncate the higher order terms over the 4th power of the stochastic field function with only an error small enough to be ignored. As expected, as the power increases the contribution form the higher order terms decreases because the stochastic field function is in the range of $-1 + \delta_f < f_\nu < 1 - \delta_f$. In comparison of Eqs. (8)–(10), it is apparent that the constants β_i, γ_i are